۱۴ آفرین — دوازدهم تجربی B — مهین ریاضی

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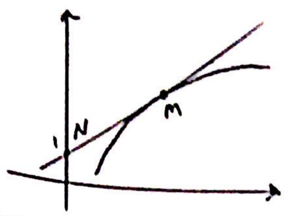

$N|^{0}_{1}$ $M|^{3}_{5}$

$m = \frac{5-1}{3-0} = \frac{4}{3} \longrightarrow f'(3) = m = \frac{4}{3}$ ②

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$f(x) = \sqrt{ax-1}$ $B|^{-1}_{1}$ $C|^{2}_{2}$ $m = \frac{2-1}{2-(-1)} = \frac{1}{3}$

معادله خط: $y-1 = \frac{1}{3}(x+1)$

معادله خط: $y = \frac{1}{3}x + \frac{4}{3}$

$\frac{1}{3}x + \frac{4}{3} = \sqrt{ax-1} \longrightarrow x+4 = 3\sqrt{ax-1}$

$x^2 + (8-9a)x + 25 = 0$

مماس = ریشه مضاعف، $\Delta = 0$

$(x \pm 5)^2 = 0 \longrightarrow 8-9a = 10$ $a = \frac{-2}{9}$ ✗

$8-9a = -10$ $a = 2$ ✓

$f(x) = \sqrt{2x-1} \xrightarrow{x=5} \sqrt{(2)(5)-1} = 3$ ②

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$y = \frac{x^2+mx+1}{x+3}$ $y' = \frac{3(m+2)}{16}$ $\frac{3(m+2)}{16} = \frac{3}{4}$ $m+2=4$ $m=2$

$4y = 3x+n \longrightarrow m = \frac{3}{4}$ $y = \frac{x^2+2x+1}{x+3} \xrightarrow{x=1} |^{1}_{1}$ $m+n = 2+1 = 3$ ②

$4 = 3+n$ $n=1$

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$f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x} = \frac{\cancel{(3-\sin x)}(9 + 3\sin x + \sin^2 x)}{\cancel{(3-\sin x)}(3+\sin x)} = \frac{9+3\sin x + \sin^2 x}{3+\sin x}$

$(3g-f)' = \left(\frac{9}{\sin x + 3} - \frac{9+3\sin x+\sin^2 x}{3+\sin x}\right)' = \left(-\left(\frac{\sin^2 x + 3\sin x}{3+\sin x}\right)\right)' = (-\sin x)'$

$(3g-f)' = -\cos x \xrightarrow{x=\frac{5\pi}{3}} -\cos\frac{5\pi}{3} = \frac{-1}{2}$ ②

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$g'(\sqrt[5]{3})\, f'(g(\sqrt[5]{3})) = (f \circ g(x))'$ $x>0$ $g(x) = \frac{1}{3x^5}$ $f(x) = \frac{-1}{\sqrt[5]{3x}}$

$f \circ g(x) = f(g(x)) = -x$ $(f \circ g)'(x) = -1$ ②

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$f(x) = \left(\frac{\sin x - 1}{\sin x + 1}\right)^2 = \left(1 + \frac{-2}{\sin x + 1}\right)^2 = 1 + \frac{4}{(1+\sin x)^2} + \frac{-4}{\sin x + 1} = 1 + x g(x)$

$x g(x) = \frac{4}{(1+\sin x)^2} - \frac{4}{\sin x+1} = \frac{-4}{\sin x+1}\left(\frac{\sin x}{\sin x+1}\right)$

$\lim_{x\to 0} g(x) = \frac{\frac{-4}{\sin x+1}\left(\frac{\sin x}{\sin x+1}\right)}{x} \xrightarrow{\sin x \sim x} \frac{-4}{(\sin x+1)^2}$ -4 ②

Scanned with CamScanner

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$$y = x^2+1 \xrightarrow{\text{قرینه نسبت به محور } x} y = -x^2-1 \longrightarrow y' = -2x$$

$A\left|\begin{matrix}\alpha\\ \beta\end{matrix}\right.$ $B\left|\begin{matrix}-\alpha\\ \beta\end{matrix}\right.$ $\quad m_1 = -2\alpha$, $m_2 = +2\alpha$ $\quad m_1 m_2 = -1$ عمود $\quad -4\alpha^2 = -1$, $\alpha = \pm\frac{1}{2}$

$A\left|\begin{matrix}\frac{1}{2}\\ \beta\end{matrix}\right.$ $B\left|\begin{matrix}\frac{-1}{2}\\ \beta\end{matrix}\right.$ $\quad y = -(\pm\frac{1}{2})^2 - 1 = \frac{-5}{4}$ ✓

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$$f(x) = 2\sqrt{x}(4x^2+3) \qquad f'(x) = \frac{1}{\sqrt{x}}(4x^2+3) + (8x)(2\sqrt{x}) = \frac{20x^2+3}{\sqrt{x}}$$

$$y - y_0 = m(x - x_0) \longrightarrow y - (2\sqrt{\alpha}(4\alpha^2+3)) = \frac{20\alpha^2+3}{\sqrt{\alpha}}(x - \alpha) \xrightarrow{(0,0)}$$

$\alpha^2 = \frac{1}{4}$ $\quad \alpha \to \frac{1}{2}$ ✓, $\frac{-1}{2}$ ✗ $\quad m = \frac{20\alpha^2+3}{\sqrt{\alpha}} = \frac{20(\frac{1}{4})+3}{\sqrt{\frac{1}{2}}} = 8\sqrt{2}$

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$$f(x) = \frac{\sqrt{x}}{-2x^2+x+1} \Rightarrow mx = \frac{\sqrt{\alpha}}{-2\alpha^2+\alpha+1} \qquad -2m\alpha^{\frac{5}{2}} + m\alpha^{\frac{1}{2}} + m\alpha^{\frac{3}{2}} = 1$$

$y = mx$

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$$(f\circ g)'(x) = g'(\tfrac{\sqrt{5}}{2})\, f'(g(\tfrac{\sqrt{5}}{2})) \qquad g(x) = \frac{1}{\sqrt{x^2-1}} \qquad f(x) = (x[x])^3$$

$$g(x) = (x^2-1)^{\frac{-1}{2}} \Rightarrow g'(x) = \frac{-1}{2}(x^2-1)^{\frac{-3}{2}} \times 2x \longrightarrow g(\tfrac{\sqrt{5}^-}{2}) = 2^+$$

$$f'(2^+) = ((2x)^3)' = (8x^3)' \longrightarrow 24x^2 = 24\times 4 = 96$$

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$$(f\circ g)'(x) = g'(\tfrac{\sqrt{5}^-}{2}) \times f'(g(\tfrac{\sqrt{5}^-}{2})) = -4\sqrt{5} \times 4\times 24 \qquad \frac{24\times 4\times(-4\sqrt{5})}{(-48\sqrt{5})}$$

خیلی … $-48\sqrt{5}$

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$$y = mx \rightarrow \frac{\sqrt{a}}{-2a^2+a+1} = ma \rightarrow \frac{1}{-2a^2+a+1} = m\sqrt{a}$$

$$m\sqrt{a}(-2a^2+a+1) = 1 \longrightarrow -2m(a^{\frac{5}{2}}) + m(a^{\frac{3}{2}}) + m(a)^{\frac{1}{2}} = 1 \xrightarrow{\text{مشتق}}$$

$$-5m(a^{\frac{3}{2}}) + \frac{3}{2}m(a^{\frac{1}{2}}) + \frac{m}{2}(a^{-\frac{1}{2}}) = 0$$

$$\frac{m}{2}(a^{-\frac{1}{2}})(-10a^2+3a+1) = 0 \rightarrow a = -\frac{1}{5} \text{ یا } a = \frac{1}{2} \checkmark \quad (a>0)$$

$$f(a) = \frac{\frac{\sqrt{2}}{2}}{-2(\frac{1}{4})+\frac{1}{2}+1} = \frac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$$