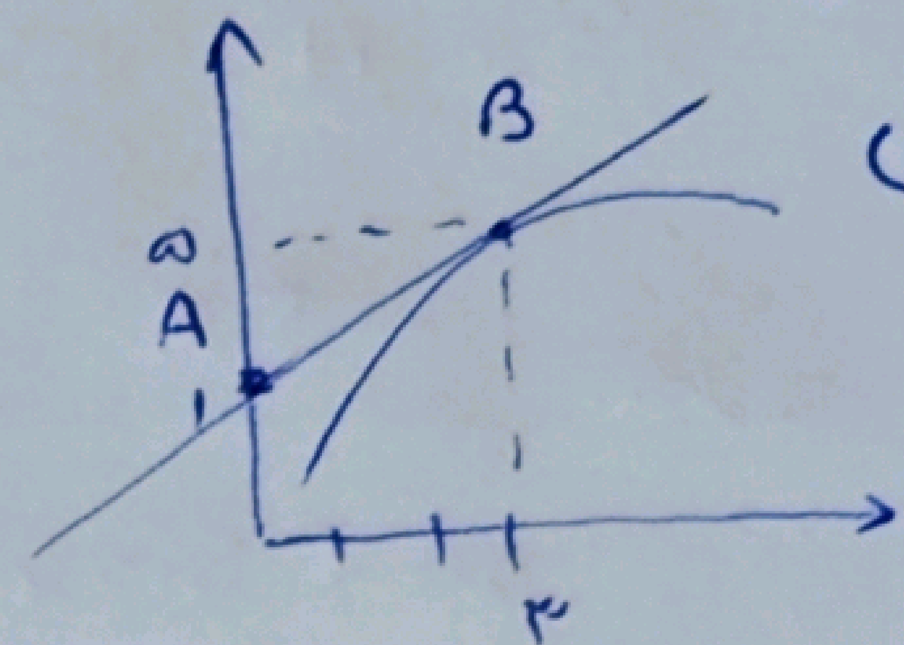
حسابان دوازدهم حضوری - B   ۱۹٫۲۵ آفرین   خورشید احمدی

۱)



$f(3)=5$, $f'(3)=\frac{y_B-y_A}{x_B-x_A}=\frac{5-1}{3-0}=\frac{4}{3}$

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۲)

$m=\frac{1}{3} \to \frac{x+4}{3}=\sqrt{ax-1} \to x^2+x(8-9a)+25=0 \xrightarrow{\text{مماس}}$

$8-9a=\pm 10 \to \boxed{a=2}$, $a=-\frac{2}{9}$

$f(5)=\sqrt{2\times 5-1}=3$

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۳)

$y'=\frac{3}{4}=\frac{(2+m)(4)-(2+m)}{16} \to \underbrace{m=2}$ ~~$n+m=3$~~

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$\frac{x^2+2m+1}{x+3} \xrightarrow{x=1} y=1 \Rightarrow y=\frac{n+3x}{4} \xrightarrow[y=1]{x=1} 1=\frac{n+3}{4} \Rightarrow \underbrace{n=1}$

۴)

$\frac{27-9\sin x-27+\sin^3 x}{9-\sin^2 x}=(3g-f)(x)$, $3g'(\frac{5\pi}{3})-f'(\frac{5\pi}{3})=(3g-f)'(\frac{5\pi}{3})$

$\Rightarrow (3g-f)(x)=-\sin x \to (3g-f)'(x)=\underbrace{-\cos x}$

$\Rightarrow (3g-f)'(\frac{5\pi}{3})=-\cos\frac{5\pi}{3}=-\frac{1}{2}$ ۲

۵)

$x>0 \to \begin{cases} f(x)=\frac{-1}{\sqrt{2x}} \\ g(x)=\frac{1}{2x^5} \end{cases} \to fog(x)=-x \Rightarrow (fog)'(x)=-1$

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$\Rightarrow (fog)'(\sqrt{3})=-1$

۶)

$$\left(\frac{\sin x-1}{\sin x+1}\right)^{\prime} = x\,g(x)+1 \rightsquigarrow \lim_{x\to 0} g(x) = \frac{-4\sin x}{(x)(\sin x+1)^2}$$

$$\sim \frac{-4\cancel{x}}{\cancel{x}(\sin x+1)^2} = -4$$

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۷)

$$y_1 = -x^2-1 \rightarrow y_1' = -2x \begin{cases} \alpha & = -2\alpha \\ -\alpha & = 2\alpha \end{cases}$$

$$\xrightarrow{\text{برهم عمودند}} -2\alpha = \frac{-1}{2\alpha} \rightarrow \alpha = \pm\frac{1}{2} \rightarrow \alpha=\frac{1}{2} \Rightarrow \boxed{y=-\frac{5}{4}}$$

فاصله d از مبدا مختصات

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۸)

$$y=mx=f(x) \Rightarrow \frac{f(x)}{x} = f'(x) \rightarrow 2(4x^2+3) = 20x^2+3$$

$$\Rightarrow x=\pm\frac{1}{2} \xrightarrow{x>0} x=\frac{1}{2} \rightarrow m = \frac{4\sqrt{2}\times 2}{1} = \boxed{8\sqrt{2}}$$

$\left|\frac{1}{2}, 4\sqrt{2}\right|$

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۹)

$$f(x)=mx \Rightarrow \frac{f(x)}{x} = f'(x) \Rightarrow \frac{\sqrt{x}}{(-2x^2+x+1)x} = \frac{4x^2-x+1}{2\sqrt{x}(-2x^2+x+1)^2}$$

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$$\Rightarrow 10x^2-3x-1=0 \rightarrow x = \begin{cases} \frac{1}{2} \\ -\frac{1}{5}\ \times \end{cases} \rightarrow \left|\begin{matrix}0\\0\end{matrix}\right., \left|\begin{matrix}\frac{1}{2}\\ \frac{\sqrt{2}}{2}\end{matrix}\right. \Rightarrow \boxed{m=\sqrt{2}}$$

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۱۰)

$$(f\circ g)'\left(\sqrt{\frac{5}{2}}\right) = \frac{-4x}{2\sqrt{(x^2-1)^3}} \cdot \left(f'\left(\frac{1}{\sqrt{x^2-1}}\right)\right) = \frac{-\sqrt{5}\times 8}{2}(2^8)(12$$

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$$\Rightarrow \frac{-48\sqrt{5}\times 2^8}{-48\sqrt{5}} = 2^8$$

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$$y = mx \longrightarrow \frac{\sqrt{a}}{-2a^2+a+1} = ma \longrightarrow \frac{1}{-2a^2+a+1} = m\sqrt{a}$$

$$m\sqrt{a}(-2a^2+a+1) = 1 \longrightarrow -2m(a^{\frac{5}{2}}) + m(a^{\frac{3}{2}}) + m(a)^{\frac{1}{2}} = 1 \xrightarrow{\text{مشتق}}$$

$$-5m(a^{\frac{3}{2}}) + \frac{3}{2}m(a^{\frac{1}{2}}) + \frac{m}{2}(a^{-\frac{1}{2}}) = 0$$

$$\frac{m}{2}(a^{-\frac{1}{2}})(-10a^2+3a+1) = 0 \longrightarrow a = -\frac{1}{5} \text{ یا } a = \frac{1}{2} \quad (a>0)$$

$$f(a) = \frac{\frac{\sqrt{2}}{2}}{-2(\frac{1}{4})+\frac{1}{2}+1} = \frac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$$

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$$g(x) = (x^2-1)^{-\frac{1}{2}} \longrightarrow g'(x) = -\frac{1}{2}(2x)(x^2-1)^{-\frac{3}{2}}$$

$$g'(\frac{\sqrt{5}}{2}) = -\frac{1}{2}(\sqrt{5})(\frac{5}{4}-1)^{-\frac{3}{2}} \longrightarrow -\frac{\sqrt{5}}{2}\,(\underbrace{\cancel{2^{-2(-\frac{3}{2})}}}_{8}) = -4\sqrt{5}$$

$$g(\frac{\sqrt{5}}{2}^-) = \frac{1}{\sqrt{\frac{5}{4}^- - 1}} = \frac{1}{\sqrt{\frac{1}{4}^-}} = \frac{1}{\frac{1}{2}^-} = 2^+$$

$$f'(2^+) = ((2x)^3)' = 24x^2 = 24 \times 4$$

$$fog'(\frac{\sqrt{5}}{2}^-) = -4\sqrt{5} \times 24 \times 4 \xrightarrow{\div -48\sqrt{5}} \frac{\overset{2}{\cancel{24}} \times \cancel{4} \times -4\sqrt{5}}{-\cancel{48}\sqrt{5}} = \boxed{8}$$