نام و نام خانوادگی ...رها بیت‌اللهی...................... پاسخنامه تشریحی تکلیف شماره .... کلاس ....

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$$g\left(\frac{\pi}{4}\right) = f(2) \to \quad g'(x) = 2\tan x\,(1+\tan^2 x) + (\sqrt{2}x - \sin x)\, f'(\tan^2 x + \sqrt{2}\cos x)$$

$$g'\left(\frac{\pi}{4}\right) = 2\times 2 + (\sqrt{2})\left(-\frac{\sqrt{2}}{2}\right)\times f'(2) \to 3f'(2) = \sqrt{3}$$

$$f'(2) = \frac{\sqrt{3}}{3}$$

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$$f(x) = \frac{\cancel{(2+\cos x)}(4 - 2\cos x + \cos^2 x)}{\cancel{(2+\cos x)}(2-\cos x)} = \frac{4 - 2\cos x + \cos^2 x}{2-\cos x} \to$$

$$f'(x) - 2g'(x) = \left(\frac{f}{g}\right)' \times g^2(x) - g'(x) = \left(\frac{4 - 2\cos x + \cos^2 x}{2}\right)' \times \frac{16}{(2+\frac{\sqrt{3}}{2})^2} - \frac{\sin x}{(2+\frac{\sqrt{3}}{2})^2}$$

$$= \frac{2\sin x + (2\cos x \times -\sin x)}{\cancel{4}} \times \frac{\cancel{16}\,4}{(2+\frac{\sqrt{3}}{2})^2} + \frac{\frac{\sqrt{3}}{2}}{(2+\frac{\sqrt{3}}{2})^2} \to \frac{\frac{-8\sqrt{3}+\sqrt{3}}{2}}{(2+\frac{\sqrt{3}}{2})^2} = \frac{-7\sqrt{3}}{2(2+\frac{\sqrt{3}}{2})^2}$$

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$$\begin{array}{c|c} & \frac{3}{4} \\ \hline (3-4x)\sqrt{ax} & (4x-3)\sqrt{ax} \end{array} \to \quad f'_+(x) = 4\sqrt{ax} + \frac{a(4x-3)}{2\sqrt{ax}}$$

$$f'_-(x) = -4\sqrt{ax} + \frac{(3-4x)a}{2\sqrt{ax}}$$

$$8\sqrt{\frac{3}{4}a} = 2\sqrt{6}$$

$$4\sqrt{\frac{3}{4}a} = \sqrt{6} \to \sqrt{12a} = \sqrt{6} \to a = \frac{1}{2}$$

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$$\left.\begin{array}{l} f'(4) = -a \\ f(4) = -4a+2 \end{array}\right\} f'(4) + f(4) = -5a + 2 = -1 \to a = \frac{3}{5}$$

$$f'(4) = -\frac{3}{5}$$

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$$y = \frac{1}{7}x + 5$$

$$f'(x) = \frac{+a+3}{(3x+1)^2} \to \frac{a+3}{(3x+1)^2} = \frac{1}{7} \to (3x+1)^2 = 7(a+3) \to 3x+1 = \sqrt{7a+21}$$

$$\frac{ax-1}{3x+1} = \frac{1}{7}x + 5 \to \frac{(a-15)x - 6}{3x+1} = \frac{1}{7}x$$

$$(3x+1)x = (a-15)x - 6$$

$$\sqrt{7a+21}$$

$$\left((a-15) - \sqrt{7a+21}\right)x = 6$$

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$$3x^2+a \to 12+a=0 \to a=-12 \to y=0$$

$$8+24-b=0 \to b=32$$

$$b-a=32-(-12)=44$$ ۶

$$۸-۲۴-b=۰ \to b=-۱۶ \to b-a=-۱۶+۱۲=-۴$$

$$\lim_{x\to 0}\frac{\sin^n(x^2)\times 2x\sin^{n-1}(x^2)\times\cos x^2}{(2\sin^2\frac{x}{2})^m} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0}\frac{(x^2)^n\times n\times(x^2)^{n-1}\times 2x}{(2(\frac{x}{2})^2)^m}=\frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2m}\times\frac{1}{2}}$$ ۷

$$=\frac{2nx^{4n-1}}{\frac{1}{2^m}\times x^{2m}}=2^{m+1}\times n\times x^{4n-2m-1}$$ پس $4n-2m-1=0 \to n=\frac{2m+1}{4}$

$$\to 2^{m+1}\times\frac{2m+1}{4}=4\sqrt{2} \to m=\frac{7}{2},\ n=2 \to 2m+n=9$$

$$x=\frac{\sqrt{y}+1}{\sqrt{y}-1}\qquad f^{-1}(2)=t \to f(t)=2 \to \sqrt{t}+1=2\sqrt{t}-2 \to \sqrt{t}=3 \to t=9$$

$$f^{-1\prime}(2)=\frac{1}{f'(9)}\ ,\quad f'(x)=\frac{-2}{(\sqrt{x}-1)^2}\times\frac{1}{2\sqrt{x}}$$ ۸

$$f'(9)=\frac{-2}{(2)^2}\times\frac{1}{6}=-\frac{1}{12} \to f^{-1\prime}(2)=-12$$

$$\frac{f(0)-f(-1)}{1}=-11 \to 1-8(1-a)=-11 \to 1-a=\frac{12}{8}=\frac{3}{2} \to a=-\frac{1}{2}$$

$$f'(-2a)=f'(1)=\underbrace{3(x^2+1)^2\,2x(-\tfrac{1}{2}x+1)}_{12-4=8}+(-\tfrac{1}{2})(x^2+1)^3$$ ۹

$$\frac{\sin\frac{\pi}{2}\cos\pi-\sin\frac{\pi}{4}\cos\frac{\pi}{2}}{\sin^4\frac{\pi}{2}-\cos^4\frac{\pi}{2}-\sin^4\frac{\pi}{4}+\cos^4\frac{\pi}{4}}=\frac{-1}{1}=-1$$ ۱۰

$$\cos x(\cancel{\cos x}-2)$$

$$f-2g\left(\frac{7\pi}{4}\right)=\frac{(2+\cancel{\cos x})(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cancel{\cos x})}-\frac{8}{2-\cos x}=\frac{\cos^2 x\ \cancel{-2}\cos x}{2-\cancel{\cos}x}=-\cos x$$

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$$(f-2g)'=-(\cos x)'=+\sin x\longrightarrow \sin\left(\frac{7\pi}{4}\right)=\boxed{-\frac{1}{2}}$$