ثنا امید قاسمی تکلیف ۲۳

۱۵

سوال ۱

$$g'_{(x)}=\left(2\,(1+\tan^2 x+1)+\sqrt{2}\,(-\sin x)\right)\times f'(\tan^2 x+\sqrt{2}\cos x)$$

$$\rightarrow g'\left(\frac{\pi}{4}\right)=(4-1)\times f'(2)=\sqrt{3}\quad\rightarrow f'(2)=\frac{\sqrt{3}}{3}=\frac{1}{\cancel{\sqrt{3}}}$$

۲

Date: Sub: ثنا ابتهاجی

سوال ۳

$f'_+(\frac{3}{4}) = +\frac{4\sqrt{3a}}{2} = 2\sqrt{3a}$

$f'_-(\frac{3}{4}) = -\frac{4\sqrt{3a}}{2} = -2\sqrt{3a}$

اختلاف $= 4\sqrt{3a} = 2\sqrt{6} \rightarrow 2\sqrt{3a} = \sqrt{6}$

$\rightarrow 3a = \frac{6}{4} \rightarrow a = \frac{1}{2}$ ✓ ۲

سوال ۴

$y = -ax + 2 \qquad f'(4) = -a \qquad f(4) = -4a + 2$

$f(4) + f'(4) = -5a + 2 = -1 \rightarrow a = \frac{3}{5} \qquad f'(4) = -a = -\frac{3}{5}$ ✓

۲

سوال ۵

$y = \frac{1}{7}x + \frac{5}{7} \qquad f(x) = \frac{ax - 1}{3x + 1}$

$f'(x) = \frac{a+3}{(3x+1)^2} = \frac{1}{7} \rightarrow 9x^2 + 6x + 1 = 7a + 21$ ۱/۵

$9x^2 + 6x - 20 - 7a = 0 \qquad \Delta = 0 \rightarrow 36 + 4 \times 9(20 + 7a) = 0$

$\rightarrow 20 + 7a = -1 \rightarrow a = -3$

سوال ۶

$f'(x) = 3x^2 + a \qquad f'(2) = 12 + a = 0 \rightarrow a = -12$

$f(2) = 0 \rightarrow 8 + 2 \times (-12) - b = 0 \rightarrow -16 - b = 0 \rightarrow b = -16$

$b - a = -4$ ✓ ۲

سوال ۸

$f^{-1}(2) = x \rightarrow f(x) = 2 \rightarrow \frac{\sqrt{x}+1}{\sqrt{x}-1} = 2 \rightarrow \sqrt{x} = 3 \rightarrow x = 9$

$f'(9) = \frac{1}{(f^{-1}(2))'}$

$f'(x) = \frac{(\frac{1}{2\sqrt{x}})(\sqrt{x}-1) - \frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2}$

$f'(9) = \frac{\frac{1}{6} \times (2) - \frac{1}{6}(4)}{4} = \frac{-2}{6 \times 4} = -\frac{1}{12} \rightarrow (f^{-1}(2))' = -12$ ✓ ۲

سوال ۹

$$\frac{f(۰) - f(-۱)}{۰+۱} = ۱ - (-۸a + ۸) = -۱۱ \to a = \frac{-۱}{۲}$$

۱,۵

$$f'(x) = ۳(۲x)(x^۲+۱)(ax+۱) + a(x^۲+۱)^۳$$

$$f'(-۱) = ۳(-۲)(\cancel{۴})\left(\frac{۳}{\cancel{۲}}\right) - \frac{۱}{۲}(۸)$$

$$-۱۸ - ۴ = -۲۲$$

سوال ۱۰

$$\frac{\sin \frac{\pi}{2} \times \overset{-1}{\cos \pi} - \sin \frac{\pi}{4} \times \overset{0}{\cos \frac{\pi}{2}}}{\frac{\pi}{4}} = \frac{-4}{\pi}$$

$$\frac{(\overset{1}{\sin^4 \frac{\pi}{2}} - \overset{0}{\cos^4 \frac{\pi}{2}}) - (\sin^4 \frac{\pi}{4} - \cos^4 \frac{\pi}{4})}{\frac{\pi}{4}} = \frac{4}{\pi}$$

$$\frac{-4/\pi}{4/\pi} = -1$$

۲

$$f-2g\left(\frac{7\pi}{4}\right) = \frac{(2+\cancel{\cos x})(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cancel{\cos x})} - \frac{8}{2-\cos x} = \frac{\overset{\cos x(\cos x-2)}{\cos^2 x-\cancel{2\cos x}}}{2-\cancel{\cos x}} = -\cos x$$

$$(f-2g)' = -(\cos x)' = +\sin x \longrightarrow \sin\left(\frac{7\pi}{4}\right) = \boxed{-\frac{1}{2}}$$

۵

$$\frac{ax-1}{3x+1} = \frac{x}{7}+\frac{5}{7} \longrightarrow 7ax-7 = 3x^2+x+15x+5$$

$$3x^2+x(16-7a)+12=0 \xrightarrow{\Delta=0} (16-7a)^2-12^2 = \underbrace{(16-7a-12)}_{a=\frac{4}{7}}\underbrace{(16-7a+12)}_{a=4}$$

$\boxed{a=4} \longrightarrow 3x^2-12x+12 \longrightarrow \boxed{x=2}$ ✓

$a=\frac{4}{7} \longrightarrow 3x^2+12x+12 \longrightarrow x=-2$ ✗ ناحیه اول

۷

$$\lim_{x\to 0}\frac{\sin^n(x^2)\times 2x\sin^{n-1}(x^2)\times\cos x^2}{\left(2\sin^2\frac{x}{2}\right)^m} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0}\frac{(x^2)^n\times n\times(x^2)^{n-1}\times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m} = \frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2m}\times\frac{1}{2}}$$

$$= \frac{2nx^{4n-1}}{\frac{1}{2^m}\times x^{2m}} = 2^{m+1}\times n\times x^{4n-2m-1}$$

پس $4n-2m-1=0 \longrightarrow n=\frac{2m+1}{4}$

پس $2^{m+1}\times\frac{2m+1}{4} = 32\sqrt{2} \longrightarrow m=\frac{7}{2},\ n=2 \longrightarrow \boxed{2m+n=9}$

۹

$$\frac{f(0)-f(-1)}{0-(-1)} = \frac{1-8(-a+1)}{1} = 8a-7=-11 \to 8a=-4 \to \boxed{a=-\frac{1}{2}}$$

$$f'(x) = 3(x^2+1)^2(2x)(ax+1)+a(x^2+1)^3 \xrightarrow{-2a=1} f'(1) = 3(2)^2(2)\left(\frac{1}{2}\right)+-\frac{1}{2}(2)^3$$

$$f'(1) = 12-4 = \boxed{8}$$