مهسا نجاتی / تکلیف شماره ی ۲۳

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$g(x)=f(\tan^2 x+\sqrt{2}\cos x)$ $\quad g'(\frac{\pi}{4})=\sqrt{2}$ $\quad f'(2)=?$

$g'(x)=2\tan x(1+\tan^2 x)$

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$$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}=\frac{\cancel{(2+\cos x)}(4-2\cos x+\cos^2 x)}{\cancel{(2+\cos x)}(2-\cos x)}$$

$$g(x)=\frac{2}{2-\cos x}\qquad f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=(f-2g)'(\frac{7\pi}{6})$$

$$\sin(\frac{7\pi}{6})=\frac{-1}{2}$$

$$f(x)-2g(x)=\frac{\cos^2 x-2\cos x}{2-\cos x}=\frac{-\cos x\,\cancel{(2-\cos x)}}{\cancel{2-\cos x}}=-\cos x\xrightarrow{\text{مشتق}}\sin x$$

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$$f(x)=|4x-3|\sqrt{ax}\qquad f'(x)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$\left|\lim_{x\to\frac{3}{4}^+}\frac{(4x-3)\sqrt{ax}-0}{x-\frac{3}{4}}-\lim_{x\to\frac{3}{4}^-}\frac{(3-4x)\sqrt{ax}-0}{x-\frac{3}{4}}\right|=2\sqrt{6}\to\left|\lim_{x\to\frac{3}{4}^+}\frac{4(x-\frac{3}{4})\sqrt{ax}}{x-\frac{3}{4}}-\lim_{x\to\frac{3}{4}^-}\frac{-4(x-\frac{3}{4})\sqrt{ax}}{x-\frac{3}{4}}\right|$$

$$=\left|4\sqrt{\frac{3a}{4}}+4\sqrt{\frac{3a}{4}}\right|=2\sqrt{6}\to\cancel{4}^{2}\sqrt{3a}=\cancel{2}\sqrt{6}\to 6=12a\to a=\frac{1}{2}$$

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$$y+ax=2\to\begin{vmatrix}4\\2-4a\end{vmatrix}\qquad f(4)+f'(4)=-1\qquad f'(4)=?\to f'(4)=-a=-\frac{3}{5}$$

$$\hookrightarrow y=-ax+2\to f'(4)=-a\qquad\hookrightarrow 2-4a-a=-1\to 3=5a\to a=\frac{3}{5}$$

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$$7y-x=5\qquad y=\frac{ax-1}{3x+1}\to y'=\frac{a+3}{(3x+1)^2}$$

$$\hookrightarrow y=\frac{x}{7}+\frac{5}{7}\to m=\frac{1}{7}$$

$$\frac{x}{7}+\frac{5}{7}=\frac{(ax-1)}{(3x+1)}\to\frac{3}{7}x^2+\frac{16}{7}x+\frac{5}{7}=ax-1\to 3x^2+(16-7a)x+12=0\xrightarrow[\text{ریشه ی مضاعف و مثبت}]{\Delta=0}$$

$$(16-7a)^2-4(3)(12)=0\to(16-7a)=144\begin{cases}16-7a=12\to a=\frac{4}{7}\\16-7a=-12\to a=4\end{cases}\xrightarrow{x=\frac{-b}{2a}}\begin{matrix}x=\frac{-12}{6}=-2\ \times\\x=\frac{+12}{6}=2\ \checkmark\end{matrix}$$

$a=4$ پس

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$f(x)=x^{۳}+ax-b \rightarrow |{}^{۲}_{۰}$  $f'(x)=۳x^{۲}+a$  $b-a=?$

$f(۲)=۸+۲a-b=۰$  $f'(۲)=۱۲+a=۰ \rightarrow a=-۱۲$

$\hookrightarrow f(۲)=۸-۲۴-b=۰ \rightarrow b=-۱۶$  $b-a=-۱۶-(-۱۲)=-۴$

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$f(x)=\sin^{۳}(x^{۲})$  $\lim\limits_{x\to ۰}\dfrac{f(x)\,f'(x)}{(۱-\cos x)^{m}}=۳۲\sqrt{۲}$  $۲m+n=?$

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$f(x)=\dfrac{\sqrt{x}+۱}{\sqrt{x}-۱} \rightarrow \sqrt{x}=-\dfrac{-y-۱}{y-۱} \rightarrow x=\left(\dfrac{y+۱}{y-۱}\right)^{۲} \rightarrow f^{-۱}(x)=\left(\dfrac{x+۱}{x-۱}\right)^{۲}$

$(f^{-۱}(x))'=۲\left(\dfrac{x+۱}{x-۱}\right)\left(\dfrac{-۲}{(x-۱)^{۲}}\right) \rightarrow (f^{-۱}(۲))'=۲(۳)(-۲)=-۱۲$

$f(x)=(x^2+1)^3(ax+1)$ $\quad f(0)=1 \quad f(-1)=8(-a+1)=-8a+8$ (9)

$$\frac{f(0)-f(-1)}{0-(-1)}=-11 \to 1+8a-8=-11 \to 8a=-4 \to a=-\frac{1}{2}$$

$$f'(x)=3(x^2+1)^2(2x)(ax+1)+(x^2+1)^3(a) \to f'(-2a)=f'(1)=$$

$$3(2)^2(2)\left(-\frac{1}{2}+1\right)+8\left(-\frac{1}{2}\right)=12-4=8$$

$y=\sin x\cos 2x$ $\quad y=\sin^4 x-\cos^4 x$ $\quad \left[\frac{\pi}{4},\frac{\pi}{2}\right] \to y=\sin^2 x-\cos^2 x=-\cos 2x$ (10)

$x=\frac{\pi}{2} \to 1\times -1=-1$ $\qquad x=\frac{\pi}{2} \to 1-0=1$

$x=\frac{\pi}{4} \to \frac{1}{2}\times 0=0$ $\qquad x=\frac{\pi}{4} \to 0$

$$\to \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1}{\frac{\pi}{2}-\frac{\pi}{4}} \qquad \to \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{1}{\frac{\pi}{2}-\frac{\pi}{4}}$$

$$\frac{\dfrac{-1}{\frac{\pi}{2}-\frac{\pi}{4}}}{\dfrac{1}{\frac{\pi}{2}-\frac{\pi}{4}}}=-1$$