مهسا نجاتی / تکلیف شماره ی ۲۳

①

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$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$ $\quad g'(\frac{\pi}{3}) = \sqrt{3}$ $\quad f'(2) = ?$

$g'(x) = 2\tan x(1+\tan^2 x)$

$g'(x) = f'(\tan^2 x + \sqrt{2}\cos x) \times (2\tan x(1+\tan^2 x) - \sqrt{2}\sin x)$

$g'(\frac{\pi}{4}) = f'(1 + \sqrt{2}\times\frac{\sqrt{2}}{2}) \times 2\times 1\times(1+1^2) - (\sqrt{2}\times\frac{\sqrt{2}}{2})$

$g'(\frac{\pi}{4}) = f'(2) \times [2\cancel{(2)} - 1] \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$

②

$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} = \frac{\cancel{(2+\cos x)}(4 - 2\cos x + \cos^2 x)}{\cancel{(2+\cos x)}(2-\cos x)}$

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$g(x) = \frac{2}{2-\cos x}$ $\quad f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = (f-2g)'(\frac{7\pi}{6})$

$\sin(\frac{7\pi}{6}) = \frac{-1}{2}$

$f(x) - 2g(x) = \frac{\cos^2 x - 2\cos x}{2-\cos x} = \frac{-\cos x\,\cancel{(2-\cos x)}}{\cancel{2-\cos x}} = -\cos x \xrightarrow{\text{مشتق}} \sin x$

③

$f(x) = |4x-3|\sqrt{ax}$ $\quad f'(x) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$\left|\lim_{x\to \frac{3}{4}^+} \frac{(4x-3)\sqrt{ax} - 0}{x-\frac{3}{4}} - \lim_{x\to\frac{3}{4}^-} \frac{(3-4x)\sqrt{ax} - 0}{x - \frac{3}{4}}\right| = 2\sqrt{6} \longrightarrow \left|\lim_{x\to\frac{3}{4}^+} \frac{4(x-\frac{3}{4})\sqrt{ax}}{x-\frac{3}{4}} - \lim_{x\to\frac{3}{4}^-} \frac{-4(x-\frac{3}{4})\sqrt{ax}}{x-\frac{3}{4}}\right|$

$= \left|4\sqrt{\frac{3a}{4}} + 4\sqrt{\frac{3a}{4}}\right| = 2\sqrt{6} \rightarrow \cancel{4}\sqrt{3a} = \cancel{2}\sqrt{6} \rightarrow 6 = 12a \rightarrow a = \frac{1}{2}$ ✓

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④

$y + ax = 2 \rightarrow \left|\begin{matrix}4\\ 2-4a\end{matrix}\right.$ $\quad f(4) + f'(4) = -1$ $\quad f'(4) = ? \longrightarrow f'(4) = -a = -\frac{3}{5}$

$\hookrightarrow y = -ax + 2 \rightarrow f'(4) = -a$ $\quad \hookrightarrow 2 - 4a - a = -1 \rightarrow 3 = 5a \rightarrow a = \frac{3}{5}$

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⑤

$7y - x = 5$ $\quad y = \frac{ax-1}{3x+1} \longrightarrow y' = \frac{a+3}{(3x+1)^2}$

$\hookrightarrow y = \frac{x}{7} + \frac{5}{7} \rightarrow m = \frac{1}{7}$

$\frac{x}{7} + \frac{5}{7} = \frac{(ax-1)}{(3x+1)} \rightarrow \frac{3}{7}x^2 + \frac{16}{7}x + \frac{5}{7} = ax - 1 \rightarrow 3x^2 + (16-7a)x + 12 = 0$ $\quad \Delta = 0$

ریشه‌ی مضاعف و مثبت

$(16-7a)^2 - 4(3)(12) = 0 \rightarrow (16-7a) = 144 \begin{cases} 16 - 7a = 12 \rightarrow a = \frac{4}{7} \\ 16 - 7a = -12 \rightarrow a = 4 \end{cases} \xrightarrow{x = \frac{-b}{2a}}$ $\quad x = \frac{-12}{6} = -2$ ✗ $\quad x = \frac{+12}{6} = 2$ ✓

$a = 4$ پس ✓

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$f(x) = x^{۳} + ax - b \rightarrow \Big|_{۰}^{۲}$ $f'(x) = ۳x^{۲} + a$ $b - a = ?$ ۶

$f(۲) = ۸ + ۲a - b = ۰$ $f'(۲) = ۱۲ + a = ۰ \rightarrow a = -۱۲$

$\hookrightarrow f(۲) = ۸ - ۲۴ - b = ۰ \rightarrow b = -۱۶$ $b - a = -۱۶ - (-۱۲) = -۴$

$f(x) = \sin^{n}(x^{۲})$ $\lim\limits_{x \to ۰} \dfrac{f(x)\, f'(x)}{(۱ - \cos x)^{m}} = ۳۲\sqrt{۲}$ $۲m + n = ?$ ۷

$\lim\limits_{x \to ۰} \dfrac{\sin^{n}(x^{۲}) \times ۲nx \sin^{n-۱}(x^{۲}) \times \cos x^{۲}}{(۲\sin^{۲}\frac{x}{۲})^{m}} \xrightarrow{\sin \approx} \lim\limits_{x \to ۰} \dfrac{(x^{۲})^{n} \times n \times (x^{۲})^{n-۱} \times ۲x}{(۲(\frac{x}{۲})^{۲})^{m}} = \dfrac{(x)^{۲n+۲n-۲+۱} \times ۲n}{(x)^{۲m} \times \frac{۱}{۲}}$

$= \dfrac{۲nx^{۴n-۱}}{\frac{۱}{۲^{m}} \times x^{۲m}} = ۲^{m+۱} \times n \times x^{۴n-۲m-۱} \rightarrow ۴n - ۲m - ۱ = ۰ \rightarrow n = \dfrac{۲m+۱}{۴}$

$\hookrightarrow ۲^{m+۱} \times \dfrac{۲m+۱}{۴} = ۳۲\sqrt{۲} \longrightarrow m = \frac{۷}{۲},\ n = ۲ \rightarrow ۲m + n = ۹$

$f(x) = \dfrac{\sqrt{x} + ۱}{\sqrt{x} - ۱} \rightarrow \sqrt{x} = -\dfrac{-y - ۱}{y - ۱} \rightarrow x = \left(\dfrac{y+۱}{y-۱}\right)^{۲} \rightarrow f^{-۱}(x) = \left(\dfrac{x+۱}{x-۱}\right)^{۲}$ ۸

$(f^{-۱}(x))' = ۲\left(\dfrac{x+۱}{x-۱}\right)\left(\dfrac{-۲}{(x-۱)^{۲}}\right) \rightarrow (f^{-۱}(۲))' = ۲(۳)(-۲) = -۱۲$

(9)

$f(x) = (x^2+1)^3(ax+1) \qquad f(0) = 1 \qquad f(-1) = 8(-a+1) = -8a+8$

$\frac{f(0) - f(-1)}{0-(-1)} = -11 \longrightarrow 1+8a-8 = -11 \rightarrow 8a = -4 \rightarrow a = -\frac{1}{2}$

$f'(x) = 3(x^2+1)^2(2x)(ax+1) + (x^2+1)^3(a) \longrightarrow f'(-2a) = f'(1) =$

$3(\cancel{2}^{4})^2(2)(-\frac{1}{2}+1) + 8(-\frac{1}{2}) = 12-4 = 8$ ✓ (2)

(10)

$y = \sin x \cos 2x \qquad y = \sin^4 x - \cos^4 x \qquad [\frac{\pi}{4}, \frac{\pi}{2}] \rightarrow y = \sin^2 x - \cos^2 x = -\cos 2x$

$x = \frac{\pi}{2} \rightarrow 1 \times -1 = -1 \qquad x = \frac{\pi}{2} \rightarrow 1 - 0 = 1$

$x = \frac{\pi}{4} \rightarrow \frac{1}{2} \times 0 = 0 \qquad x = \frac{\pi}{4} \rightarrow 0$

$\rightarrow \frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{-1}{\frac{\pi}{2} - \frac{\pi}{4}} \qquad \rightarrow \frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{1}{\frac{\pi}{2} - \frac{\pi}{4}} \qquad \frac{\frac{-1}{\frac{\pi}{2} - \frac{\pi}{4}}}{\frac{1}{\frac{\pi}{2} - \frac{\pi}{4}}} = -1$ ✓ (2)