تکلیف سری ۲۳ — بسمه تعالی — کریمی، دوازدهم دختر B

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①

$$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$$

$$g'(x) = \left(2\tan x(1+\tan^2 x) + -\sqrt{2}\sin x\right)\left(f'(\tan^2 x + \sqrt{2}\cos x)\right)$$

$$g'\left(\frac{\pi}{4}\right) = \left(2(2) + (-1)\right) f'(2) = \sqrt{3} \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$$

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②

$$f(x) = \frac{\cos^3 x + 8}{4-\cos^2 x} = \frac{\cancel{(\cos x+2)}(\cos^2 x - 2\cos x + 4)}{(2-\cos x)\cancel{(2+\cos x)}} \Rightarrow f'(x) = \frac{(2\cos x\sin x + 2\sin x) - (\sin x)(\cdots)}{(2-\cos x)^2}$$

$$g(x) = \frac{2}{2-\cos x} \longrightarrow g'(x) = \frac{0-(\sin x)}{(2-\cos x)^2} = \frac{-\sin x}{(2-\cos x)^2}$$

$$f'\left(\frac{7\pi}{6}\right) - 2g'\left(\frac{7\pi}{6}\right) = \frac{36-96\sqrt{3}}{169} - 2\left(\frac{4(19)-32\sqrt{3}}{169}\right) = \frac{36-96\sqrt{3}-152+64\sqrt{3}}{169}$$

$$= \frac{-116}{169}$$

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③

$$\left|f'_+\left(\frac{3}{4}\right) - f'_-\left(\frac{3}{4}\right)\right| = 2\sqrt{6}$$

$$f_+\left(\frac{3}{4}\right) = (4x-3)\sqrt{ax} \Rightarrow f'_+\left(\frac{3}{4}\right) = 4(\sqrt{ax}) + \frac{a}{2\sqrt{ax}}(4x-3) = 4\sqrt{\frac{3a}{4}}$$

$$f'_-\left(\frac{3}{4}\right) = -4\sqrt{\frac{3a}{4}} \Rightarrow 8\sqrt{\frac{3a}{4}} = 2\sqrt{6}$$

$$\sqrt{3a\times 16} = \sqrt{24} \longrightarrow 3a\times\cancel{16}^{4} = \cancel{24}^{6}$$

$$3a = \frac{3}{2} \rightarrow a = \frac{1}{2}$$

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④

$$y + ax = 2 \rightarrow y = -ax+2$$

$$f(4) + f'(4) = -1 \rightarrow -4a+2+-a = -1 \rightarrow -5a+2 = -1$$

$$f'(4) = ?$$

$$-5a = -3 \rightarrow a = \frac{3}{5}$$

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$7y - x = 5 \longrightarrow y = \frac{x+5}{7}$

$y = \frac{ax-1}{3x+1}$

$\longrightarrow \frac{1}{7} = \frac{a+3}{(3x+1)^2} \longrightarrow (3x+1)^2 = 7(a+3)$

$\longrightarrow 7a = 9x^2 + 6x - 20$

$\frac{ax-1}{3x+1} = \frac{x+5}{7} \longrightarrow 7a = 9x^2 + 6x - 20$

$7ax - 7 = 3x^2 + 16x + 5$

$9x^3 + 6x^2 - 20x - 7 = 3x^2 + 16x + 5$

$9x^3 + 3x^2 - 36x - 12 = 0$

$(x^2 - 4)(3x+1) = 0 \Rightarrow x = -2$ غ‌ق‌ق

$x = \frac{-1}{3}$ غ‌ق‌ق

$x = 2$ ✓

$x = 2 \longrightarrow 7a = 9(4) + 12 - 20$

$7a = 28 \longrightarrow a = 4$ (درست است)

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$f(x) = x^3 + ax - b$

$f(2) = 0 \longrightarrow 8 + 2a - b = 0 \longrightarrow 2a - b = -8$

$f'(2) = 0 \longrightarrow 3x^2 + a = 12 + a = 0 \longrightarrow a = -12$

$\Big\} \to b = -16$

$b - a = -16 + 12 = -4$

$$f'(x) = n \sin^{n-1}(x^2) \cdot 2x \cdot \cos(x^2)$$

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$$\lim_{x\to 0} \frac{f(x) f'(x)}{(1-\cos x)^m} = \frac{\sin^n(x^2) \cdot 2nx \cdot \sin^{n-1}(x^2) \cdot \cancel{\cos x^2}^{\,\to 1}}{(1-\cos x)^m} = \frac{(x^2)^{2n-1} \times 2nx \times 1}{(\frac{1}{2} x^2)^m}$$

$$\frac{x^{4n-2} \times 2nx \times 2^m}{x^{2m}} = \frac{n \times 2^{m+1} \times x^{4n-1}}{x^{2m}} = ۳۲\sqrt{۲}$$

$4n-1 = 2m$ $\quad n = 2$ $\quad 2m+n = 7+2 = 9$

$n \times 2^{m+1} = 2^{\frac{11}{2}}$ $\quad m = \frac{7}{2}$

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$$F^{-1}(x) = y = \left(\frac{x+1}{x-1}\right)^2 \Rightarrow y'(2) = 2\left(\frac{x+1}{x-1}\right)\left(\frac{(x-1)-(x+1)}{(x-1)^2}\right) = 2(3)(-2) = -12$$

ج آخر

$f(x) = (x^2+1)^3(ax+1)$ ⑨

$[-1, 0]$

آهنگ متوسط: $\dfrac{f(0) - f(-1)}{0 - (-1)} = 8a - 7 = -11 \rightarrow a = \dfrac{-1}{2}$

$F'(1) = ?$

$F'(x) = 3(x^2+1)^2(2x)\left(\dfrac{-1}{2}x+1\right) + \dfrac{-1}{2}(x^2+1)^3$

$F'(1) = 3(4)(2)\left(\dfrac{1}{2}\right) + \dfrac{-1}{2}(8) = 8$

$y = \sin x \cdot \cos 2x$ ⑩

$y' = \sin^2 - \cos^4 x$

آهنگ متوسط y $= \dfrac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{-1 - \frac{\sqrt{2}}{2} \times 0}{\frac{\pi}{4}} = \dfrac{-4}{\pi}$

آهنگ متوسط y' $= \dfrac{y'(\frac{\pi}{2}) - y'(\frac{\pi}{4})}{\frac{\pi}{4}} = \dfrac{1}{\frac{\pi}{4}} = \dfrac{4}{\pi}$

$\dfrac{\frac{-4}{\pi}}{\frac{4}{\pi}} = -1$

$^{n-1}(2)\ 2x\ \cos(x^2)$