تکلیف سری ۲۳ — بسمه تعالی (بنام خدا) — دوازدهم دختر B

۱۹٫۵ آفرین

(۱)

$g(x) = f(\tan^2 x + \sqrt{2}\cos x)$

$g'(x) = \left(2\tan x(1+\tan^2 x) + -\sqrt{2}\sin x\right)\left(f'(\tan^2 x + \sqrt{2}\cos x)\right)$

$g'(\frac{\pi}{4}) = \left(2(2) + (-1)\right) f'(2) = \sqrt{3} \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$

۲

(۲)

$f(x) = \frac{\cos^3 x + 8}{4 - \cos^2 x} = \frac{\cancel{(\cos x + 2)}(\cos^2 x - 2\cos x + 4)}{(2-\cos x)\cancel{(2+\cos x)}} \Rightarrow f'(x) = \frac{(2\cos x\sin x + 2\sin x) - (\sin x)(\ldots)}{(2-\cos x)^2}$

$g(x) = \frac{2}{2-\cos x} \longrightarrow g'(x) = \frac{0 - (\sin x)}{(2-\cos x)^2} = \frac{-\sin x}{(2-\cos x)^2}$

۱٫۵

$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = \frac{36 - 64\sqrt{3}}{169} - 2\left(\frac{4(19) - 32\sqrt{3}}{169}\right) = \frac{36 - 64\sqrt{3} - 152 + 64\sqrt{3}}{169}$

$= \frac{-116}{169}$

(۳)

$\left|f'_+(\frac{3}{4}) - f'_-(\frac{3}{4})\right| = 2\sqrt{6}$

$f_+(\frac{3}{4}) = (4x-3)\sqrt{ax} \Rightarrow f'_+(\frac{3}{4}) = 4(\sqrt{ax}) + \frac{a}{2\sqrt{ax}}(4x-3) = 4\sqrt{\frac{3a}{4}}$

$f'_-(\frac{3}{4}) = -4\sqrt{\frac{3a}{4}} \Rightarrow 8\sqrt{\frac{3a}{4}} = 2\sqrt{6}$

$\sqrt{3a \times 16} = \sqrt{24} \longrightarrow 3a \times \cancel{16} = \cancel{24}$ (4, 6)

$3a = \frac{3}{2}$

$a = \frac{1}{2}$

۲

(۴)

$y + ax = 2 \longrightarrow y = -ax + 2$

$f(4) + f'(4) = -1 \longrightarrow -4a + 2 + -a = -1 \longrightarrow -5a + 2 = -1$

$f'(4) = ?$

$-5a = -3 \longrightarrow a = \frac{3}{5}$

۲

۵

$$7y - x = 5 \longrightarrow y = \frac{x+5}{7}$$

$$y = \frac{ax-1}{3x+1} \longrightarrow \frac{1}{7} = \frac{a+3}{(3x+1)^2} \longrightarrow (3x+1)^2 = 7(a+3)$$

$$\longrightarrow 7a = 9x^2 + 6x - 20$$

$$\frac{ax-1}{3x+1} = \frac{x+5}{7} \longrightarrow 7a = 9x^2 + 6x - 20$$

$$7ax - 7 = 3x^2 + 16x + 5$$

$$9x^3 + 6x^2 - 20x - 7 = 3x^2 + 16x + 5$$

$$9x^3 + 3x^2 - 36x - 12 = 0$$

$$(x^2 - 4)(3x+1) \Rightarrow x = -2 \quad \text{غ ق ق}, \quad x = \frac{-1}{3} \quad \text{غ ق ق}, \quad x = 2 \quad \checkmark$$

۲

$$x = 2 \longrightarrow 7a = 9(4) + 12 - 20$$

$$7a = 28 \longrightarrow a = 4$$

درنتیجه اول ()

۶

$$f(x) = x^3 + ax - b$$

$$f(2) = 0 \longrightarrow 8 + 2a - b = 0 \longrightarrow 2a - b = -8$$

$$f'(2) = 0 \longrightarrow 3x^2 + a = 12 + a = 0 \longrightarrow a = -12$$

$$b = -16$$

۲

$$b - a = -16 + 12 = -4$$

$$f'(x) = n\sin^{n-1}(x^۲)\cdot ۲x\cdot \cos(x^۲)$$

۷

$$\lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m} = \frac{\sin^n(x^۲)\cdot ۲nx\cdot \sin^{n-1}(x^۲)\cdot \cancel{\cos x}^{\,1}}{(1-\cos x)^m} = \frac{(x^۲)^{۲n-1}\times ۲nx \times 1}{(\frac{1}{۲}x^۲)^m}$$

$$\frac{x^{۴n-۲}\times ۲nx\times ۲^m}{x^{۲m}} = \frac{n\times ۲^{m+1}\times \cancel{x^{۴n-1}}}{\cancel{x^{۲m}}} = ۳۲\sqrt{۲}$$

ب

$$۴n-1 = ۲m \qquad n=۲ \qquad ۲m+n = ۷+۲ = ۹$$

$$n\times ۲^{m+1} = ۲^{\frac{11}{۲}} \qquad m = \frac{۷}{۲}$$

۸

$$f^{-1}(x) = y = \left(\frac{x+1}{x-1}\right)^۲ \Rightarrow y'(۲) = ۲\left(\frac{x+1}{x-1}\right)\left(\frac{x-1-(x+1)}{(x-1)^۲}\right) = ۲(۳)(-۲) = -1۲$$

جواب

ب

$f(x) = (x^۲+۱)^۳(ax+۱)$ $\qquad$ ۹

$[-۱, ۰]$

$\text{آهنگ متوسط} = \frac{f(۰) - f(-۱)}{۰ - (-۱)} = ۸a - ۷ = -۱۱ \rightarrow a = \frac{-۱}{۲}$

۲

$f'(۱) = ?$

$f'(x) = ۳(x^۲+۱)^۲(۲x)(\frac{-۱}{۲}x+۱) + \frac{-۱}{۲}(x^۲+۱)^۳$

$f'(۱) = ۳(۴)(۲)(\frac{۱}{۲}) + \frac{-۱}{۲}(۸) = ۸$

۱۰

$y = \sin x \cdot \cos ۲x$

$y' = \sin^۴ - \cos^۴ x$

$\text{آهنگ متوسط } y = \frac{f(\frac{\pi}{۲}) - f(\frac{\pi}{۴})}{\frac{\pi}{۲} - \frac{\pi}{۴}} = \frac{-۱ - \frac{\sqrt{۲}}{۲} \times ۰}{\frac{\pi}{۴}} = \frac{-۴}{\pi}$

۲

$\text{آهنگ متوسط } y' = \frac{y'(\frac{\pi}{۲}) - y'(\frac{\pi}{۴})}{\frac{\pi}{۴}} = \frac{۱}{\frac{\pi}{۴}} = \frac{۴}{\pi}$

$\frac{\frac{-۴}{\pi}}{\frac{۴}{\pi}} = -۱$

$n-۱ \quad (\quad ۲) \quad ۲x \quad \cos(x^۲)$

$$f-2g\left(\frac{7\pi}{4}\right)=\frac{(2+\cancel{\cos x})(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cancel{\cos x})}-\frac{4}{2-\cos x}=\frac{\overset{\cos x(\cancel{\cos x}-2)}{\cos^2 x-2\cos x}}{2-\cancel{\cos x}}=-\cos x$$

۲

$$(f-2g)'=-(\cos x)'=+\sin x\longrightarrow \sin\left(\frac{7\pi}{4}\right)=\boxed{-\frac{1}{2}}$$