ملینا کبیری ۱۸

Subject: ( ) Date: ....

۱)

$g'(x)=۲(\tan x)(1+\tan^۲ x)-\sqrt{۲}\sin x \times f'(\tan^۲ x+\sqrt{۲}\cos x)$

$g'(\frac{\pi}{۴})=۲\times f'(۲)-\sqrt{۳}\to f'(۲)=\frac{\sqrt{۳}}{۳}$ ۲

۲)

$f(x)=\frac{۲+\cos}{۲+\cos}\times\frac{۴-۲\cos+\cos^۲}{۲-\cos}$ $g(x)=\frac{۲}{۲-\cos}$

$f'(\frac{۷\pi}{۶})-۲g'(\frac{۷\pi}{۶})=(f-۲g)'(\frac{۷\pi}{۶})$

$f-۲g=\frac{\cos^۲-۲\cos x+۴-۴}{۲-\cos}=-\cos\to\sin x$ ۲

$(f-۲g)'(\frac{۷\pi}{۶})=\sin\frac{۷\pi}{۶}=-\frac{۱}{۲}$

۳)

$A\ x>\frac{۳}{۴}\to(۴x-۳)\sqrt{ax}=۴\sqrt{ax}$

$A\ x<\frac{۳}{۴}\to(-۴x+۳)\sqrt{ax}=-۴\sqrt{ax}$ ۲

$A-A=۲(۴\sqrt{ax})=۲\sqrt{۶}\xrightarrow{÷۲}\sqrt{۶}\to a=\frac{۱}{۲}$

۴)

$\to y=۲-ax$ $f(۴)=۲-۴a$

$f'(۴)=a\to f(۴)+f'(۴)=۲-۴a$ $a-۲=۵a$ $a=\frac{۲}{۵}$

۲ $f(۴)=-\frac{۳}{۵}$

۵)

$\frac{۱}{۷}=\frac{a+۳}{(۳x+۱)^۲}\Rightarrow a=\frac{(۳x+۱)^۲-۲۱}{۷}$

$\frac{d+x}{۷}=\frac{ax-۱}{۳x+۱}\to\frac{d+x}{۷}=\frac{x(۳x+۱)^۲-۲۱x-۷}{۷(۳x+۱)}$

$۹x^۳+۳x^۲-۲۴x-۱۲=0\to(x^۲-۴)(۳x+۱)=0$ ۲

$x=۲$ ناحیه ۱

$a=۴$

Arman

Subject: (   )   Date: ....

$f'(x) = 3x^2 + a \xrightarrow{(2,0)} 12 + a = 0 \to a = -12$ 4

2

$f(x) = x^3 - 12x - b \xrightarrow{(2,0)} 8 - 24 - b = 0 \to b = -16$

$b - a = -4$

$\dfrac{\sqrt{x}+1}{\sqrt{x}-1} \to \dfrac{\sqrt{y}+1}{\sqrt{y}-1} \to y = \left(\dfrac{x+1}{x-1}\right)^2$ 8

2

$(f^{-1}(x))' = \left(\left(\dfrac{x+1}{x-1}\right)^2\right)' = 2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{-2}{(x-1)^2}\right) \to 2\times 3\times -2$

-12

$\dfrac{f(0) - f(-1)}{0+1} = 1 - 8(-a+1) \Rightarrow a = \dfrac{1}{2}$ 9

3

$f'(1) = 3(x^2+1)^2(2x)\left(\dfrac{-x}{2}+1\right) - \dfrac{1}{2}(x^2+1)^3 = \boxed{8}$

$y = \sin\cos\pi x$ 10

$\lim \dfrac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{-1-0}{\frac{\pi}{4}} = \dfrac{-4}{\pi}$

2

$y = \sin^4 - \cos^4 = (\sin^2 - \cos^2)(\sin^2 + \cos^2)$

$\lim \dfrac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \dfrac{4}{\pi} = \boxed{-1}$

7

$$\lim_{x\to 0}\frac{\sin^{n}(x^{2})\times 2x\sin^{n-1}(x^{2})\times \cos x^{2}}{\left(2\sin^{2}\frac{x}{2}\right)^{m}} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0}\frac{(x^{2})^{n}\times n\times (x^{2})^{n-1}\times 2x}{\left(2\left(\frac{x}{2}\right)^{2}\right)^{m}} = \frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2m}\times \frac{1}{2}}$$

$$= \frac{2nx^{4n-1}}{\frac{1}{2^{m}}\times x^{2m}} = 2^{m+1}\times n\times x^{4n-2m-1} \Rightarrow 4n-2m-1=0 \rightarrow n=\frac{2m+1}{4}$$

$$\Rightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2} \longrightarrow m=\frac{7}{2},\ n=2 \rightarrow \boxed{2m+n=9}$$