پارمیس زندی

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سؤال ۱

$g(x)=f(\tan^2 x+\sqrt{2}\cos x)$

$g'(\frac{\pi}{4})=\sqrt{3}$

$f'(2)=?$

$g'(x)=f'(\tan^2 x+\sqrt{2}\cos x)\times 2\tan x(1+\tan^2 x)-\sqrt{2}\sin x)$

$g'(\frac{\pi}{4})=f'(1+\sqrt{2}\times\frac{\sqrt{2}}{2})\times 2\times 1\times(1+1^2)-(\sqrt{2}\times\frac{\sqrt{2}}{2})$

$g'(\frac{\pi}{4})=f'(2)\times[2\times 2-1]\rightarrow f'(2)=\frac{\sqrt{3}}{3}$

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سؤال ۲

$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}$ $g(x)=\frac{2}{2-\cos x}$ $f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=?$

$f'(x)$

$f-2g(\frac{7\pi}{6})=\frac{(2+\cos x)(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cos x)}-\frac{4}{2-\cos x}=\frac{\cos^2 x-2\cos x}{2-\cos x}=-\cos x$

$(f-2g)'=-(\cos x)'=+\sin x\rightarrow \sin(\frac{7\pi}{6})=-\frac{1}{2}$

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سؤال ۳

$f(x)=|4x-3|\sqrt{ax}$ $x=\frac{3}{4}$ $2\sqrt{9}$ $a=?$

اگر $x>\frac{3}{4}\Rightarrow(4x-3)\sqrt{ax}=f(x)\Rightarrow f'_{+}(x)=\frac{\sqrt{a}}{2\sqrt{x}}(4x-3)+4\sqrt{ax}\Rightarrow f'_{+}(\frac{3}{4})=2\sqrt{3a}$

اگر $x<\frac{3}{4}\Rightarrow f(x)=(-4x+3)\sqrt{ax}\Rightarrow f'_{-}(x)=\frac{\sqrt{a}}{2\sqrt{x}}(3-4x)-4\sqrt{ax}\Rightarrow f'_{-}(\frac{3}{4})=-2\sqrt{3a}$

$2\sqrt{3a}-(-2\sqrt{3a})=4\sqrt{3a}=2\sqrt{9}\Rightarrow \boxed{a=\frac{1}{3}}$

سؤال ۴

$y=ax+2$ $f(4)+f'(4)=-1$ $f'(4)=?$

$x=4$

$y'=a$ $f(4)=4a+2\Rightarrow(4a+2)+a=-1\Rightarrow 5a+2=-1\Rightarrow 5a=-3\Rightarrow$

$a=-\frac{3}{5}\Rightarrow \boxed{f'(4)=-\frac{3}{5}}$

سؤال ۵

$$7y - x = 5 \Rightarrow 7y = 5 + x \Rightarrow y = \frac{x+5}{7}$$

$$y = \frac{ax-1}{3x+1} \Rightarrow \frac{x+5}{7} = \frac{ax-1}{3x+1} \Rightarrow (3x+1)(x+5) = (ax-1)7 \Rightarrow$$

$a = ?$

$$3x^2 + 16x + 5 = 7ax - 7 \Rightarrow \underset{a}{3}x^2 + (\underset{b}{16 - 7a})x + \underset{c}{12} = 0$$

ریشه مضاعف دارد، $\Delta = 0$ پس

$$\Rightarrow 3(x-2)^2 = 0 \,,\; 16 - 7a = -12 \Rightarrow 7a = 28 \Rightarrow \boxed{a = 4}$$

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سؤال ۶

$f(x) = x^3 + ax - b \qquad f(2) = 0$

$x = 2$

$b - a = ?$

$$f'(x) = 3x^2 + a \Rightarrow f'(2) = 3(2)^2 + a = 12 + a = 0 \Rightarrow \boxed{a = -12}$$

$$f(2) = 8 + 2(-12) - b = 0 \Rightarrow \boxed{b = -16}$$

$$b - a = -16 - (-12) = \boxed{-4}$$

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سؤال ۷

$f(x) = \sin^n(x^2)$

$$\lim_{x \to 0} \frac{f(x) \cdot f'(x)}{(1-\cos x)^m}$$

$2m + n = ?$

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سؤال ۸

$$f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow y = \frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow y\sqrt{x} - y = \sqrt{x} + 1 \Rightarrow \sqrt{x}\,y - \sqrt{x} = y + 1$$

$f^{-1}(x)$

$$\Rightarrow \sqrt{x}(y-1) = y+1 \rightarrow \sqrt{x} = \frac{y+1}{y-1} \Rightarrow x = \left(\frac{y+1}{y-1}\right)^2 \rightarrow f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2$$

$x = 2$

$$f^{-1'}(x) = 2\left(\frac{x+1}{x-1}\right)\left(\frac{1(x-1) - 1(x+1)}{(x-1)^2}\right) \underset{x=2}{\overset{\text{جایگذاری}}{\Rightarrow}} -12$$

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⑨ مسئله

$f(x)=(x^2+1)^2(ax+1) \rightsquigarrow \frac{f(0)-f(-1)}{0-(-1)}=\frac{1-(8(1-a))}{1}=8a-7=-11 \Rightarrow a=-\frac{1}{2}$

$[-1,0]$

-11

$x=-2a$

$\Rightarrow f(x)=(x^2+1)^2(-\frac{1}{2}x+1) \Rightarrow f'(x)=2(2x)(x^2+1)^1(-\frac{x}{2}+1)-\frac{1}{2}(x^2+1)^2$

$\Rightarrow \boxed{f'(1)=8}$ ✓

$y=\sin x\cos 2x \Rightarrow \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{\sin\frac{\pi}{2}\cos\pi-\sin\frac{\pi}{4}\cos\frac{\pi}{4}}{\frac{\pi}{4}}=-\frac{4}{\pi}$ ① ⑩ مسئله

$y=\sin^4 x-\cos^4 x$

تجزیه ✓

$(\underbrace{\sin^2 x+\cos^2 x}_{①})(\sin^2 x-\cos^2 x)=\sin^2 x-\cos^2 x=-\cos 2x$

$\Rightarrow \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-\cos\pi+\cos\frac{\pi}{2}}{\frac{\pi}{4}}=\frac{4}{\pi}$ ②

$\frac{①}{②}=\frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=\boxed{-1}$

$\lim_{n\to 0} \frac{\sin^n(x^2)\times 2n\sin^{n-1}(x^2)\times\cos x^2}{(2\sin\frac{x}{2})^n} \overset{\text{هم ارز}}{\rightsquigarrow} \lim_{n\to 0}\frac{(x^2)^n\times n\times(x^2)^{n-1}\times 2n}{(2(\frac{x}{2})^2)^n}=\frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2n}\times\frac{1}{2}}$ ✓

$=\frac{2nx^{4n-1}}{\frac{1}{2^n}x^{2n}}=2^{n+1}\times n\times x^{4n-2n-1}$ ↳ $4n-2n-1\neq 0 \rightarrow n=\frac{2n+1}{4}$

↳ $2^{n+1}\times\frac{2n+1}{4}=2\sqrt{2} \rightarrow m=\frac{1}{2}, n=2 \rightarrow 2m+n=9$