۲۰ آفرین

باران تسلیمی

۱ـ $g(x) = f(\tan^2 x + \sqrt{2}\cos x)$ و $g'(\frac{\pi}{4}) = \sqrt{3}$ مقدار $f'(2) = ?$

$$g'(\frac{\pi}{4}) = \underbrace{(2\tan x + 2\tan^3 x - \sqrt{2}\sin x)}_{3} \times f'(\tan^2 x + \sqrt{2}\cos x) = \sqrt{3}$$

$$3 \times f'(2) = \sqrt{3}$$

$$\Rightarrow f'(2) = \frac{\sqrt{3}}{3}$$

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$$f(x) = \frac{8 + \cos^3 x}{4 - \cos^2 x} \Rightarrow \frac{(2 + \cos x)(\cos^2 x - 2\cos x + 4)}{(2 + \cos x)(2 - \cos x)} = \frac{\cos^2 x - 2\cos x + 4}{2 - \cos x}$$

$$g(x) = \frac{2}{2 - \cos x}$$

$$f'\left(\frac{7\pi}{4}\right) - 2g'\left(\frac{7\pi}{4}\right) = (f - 2g)'\left(\frac{7\pi}{4}\right)$$

$$\hookrightarrow \frac{\cos^2 x - 2\cos x + 4}{2 - \cos x} - \frac{4}{2 - \cos x} = \frac{\cos^2 x - 2\cos x}{2 - \cos x} \rightarrow \frac{-\cos x(2 - \cancel{\cos x})}{2 - \cancel{\cos x}} = -\cos x$$

$$(f - 2g)' = \sin x \xrightarrow{x = \frac{7\pi}{4}} \boxed{\frac{-1}{2}}$$

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$$f(x) = |4x - 3|\sqrt{ax} \qquad \text{شیب} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'_+\left(\frac{3}{4}\right) = \lim \frac{|4x - 3|\sqrt{ax}}{x - \frac{3}{4}} \xrightarrow{\frac{3}{4}^+} \frac{(4x - 3)\sqrt{ax}}{x - \frac{3}{4}} = \frac{4\cancel{(x - \frac{3}{4})}\sqrt{ax}}{\cancel{x - \frac{3}{4}}} = 4\sqrt{ax}$$

$$f'_-\left(\frac{3}{4}\right) = \lim \frac{-(4x - 3)\sqrt{ax}}{x - \frac{3}{4}} = -4\sqrt{ax}$$

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$$4\sqrt{ax} - (-4\sqrt{ax}) \Rightarrow 8\sqrt{ax} = 2\sqrt{6} \rightarrow \sqrt{ax} = \frac{\sqrt{6}}{4} \xrightarrow{\text{توان ۲}} ax = \frac{\cancel{6}^{\,3}}{\cancel{16}_{\,8}}$$

$$8ax = 3 \rightarrow x = \frac{3}{4} \rightsquigarrow \cancel{8}^{2} \times \frac{3}{\cancel{4}} \times a = 3 \rightarrow 2a = 1 \rightarrow \boxed{a = \frac{1}{2}}$$

۴- $y+ax=2$ در نقطه‌ای به طول ۴ بر تابع $f$ مماس است، $f(4)+f'(4)=-1$

$f'(4)=?$

$$y=-ax+2$$

$$-a=f'(4)$$

$$f(4)=-4a+2$$

$$\Rightarrow -5a+2=-1$$

$$-5a=-3 \longrightarrow a=\frac{3}{5}$$

$$f'(4)=\frac{-3}{5}$$

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۵- $7y-x=5$ در ناحیه اول بر $y=\frac{ax-1}{3x+1}$ مماس است $a$؟

$$7y=x+5 \longrightarrow y=\underbrace{\frac{1}{7}}_{\text{شیب}}x+\frac{5}{7}$$

$$f'(x)=\frac{a+3}{(3x+1)^2}=\frac{1}{7}$$

$$\frac{x+5}{7}=\frac{ax-1}{3x+1} \longrightarrow 3x^2+x+15x+5=7ax-7$$

$$3x^2+16x-7ax+12=0 \xrightarrow[\text{(ریشه مضاعف دارد)}]{\text{جمع عامل}} 3(x-2)^2$$

$$16-7a=-12 \longrightarrow \boxed{a=4}$$

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$f(2)=0 \;,\; f'(2)=0$ $\quad \left[ \begin{array}{l} f(2)=0 \rightarrow 8+2a-b=0 \quad b=-16 \\ f'(2)=0 \rightarrow 12+a=0 \rightarrow a=-12 \end{array} \right.$ ۶-

$f'(x)=3x^2+a$

$b-a=-4$ ۲

هم‌ارزی $\rightarrow \lim_{x\to 0} \dfrac{(x^2)^n (x^2)^{n-1} \times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m} =$ ۷-

$\lim_{x\to 0} \dfrac{2n x^{4n-1}}{2^{-m} x^{2m}} = \lim_{x\to 0} 2^{m+1} n\, x^{4n-2m-1} \rightarrow$ حاصل حد عددی غیر متناهی و غیر صفر (توان صفر) ۲

$m=\dfrac{7}{2} \;,\; n=2 \longrightarrow 2m+n=9$

$f^{-1}(x)=\left(\dfrac{x+1}{x-1}\right)^2 \xrightarrow{x=2} 2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{(x-1)-(x+1)}{(x-1)^2}\right) \xrightarrow{[illegible]}$ ۸-

$\Rightarrow -12$ ۲

$\dfrac{f(0)-f(-1)}{0-(-1)} = \dfrac{1-(8(1-a))}{1}$ ۹-

$8a-7=-11 \longrightarrow a=\dfrac{-1}{2}$

$f'(x)=3(2x)(x^2+1)^2\left(-\frac{x}{2}+1\right)-\frac{1}{2}(x^2+1)^3$

$f(x)=(x^2+1)^3\left(\frac{-x}{2}+1\right)$ $\quad \hookrightarrow f'(1)=8$ ۲

$y=\sin x\cos 2x \Rightarrow \dfrac{y(\frac{\pi}{2})-y(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}} = \dfrac{\sin\frac{\pi}{2}\cos\pi-\sin\frac{\pi}{4}\cos\frac{\pi}{2}}{\frac{\pi}{4}}$ ۱۰-

$\Rightarrow \dfrac{1\times(-1)-0}{\frac{\pi}{4}} = \dfrac{-4}{\pi}$

$y=\sin^4 x-\cos^4 x=(\sin^2 x+\cos^2 x)\underbrace{(\sin^2 x-\cos^2 x)}_{-\cos 2x}$

$\dfrac{y(\frac{\pi}{2})-y(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}} \Rightarrow -1$ ۲