۱۴ سه رازی

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$g'(x) = (2\tan x)(1+\tan^2 x) - \sqrt{2}\sin x)\, f'(\tan^2 x + \sqrt{2}\cos x)$

$g'(\frac{\pi}{4}) = (2\times 2 - 1) f'(1+1)$ $\sqrt{3} = 3f'(2)$ $f'(2) = \frac{\sqrt{3}}{3}$ ۴

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$f(x) - 2g(x) = \frac{1+\cos^3 x}{2-\cos^2 x} - \frac{2}{2-\cos x} = \frac{1+\cos^3 x - 1 - 2\cos x}{2-\cos^2 x} = -\cos x$ ۲

$f'(x) - 2g'(x) = \sin x$ $f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = \sin(\frac{7\pi}{6}) = -\frac{1}{2}$

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$x > \frac{3}{2} \to f(x) = (2x-3)\sqrt{ax} \to f'(x) = 2\sqrt{ax}$ $f'_+(\frac{3}{2}) = 2\sqrt{\frac{3a}{2}} = \sqrt{6a}$

$x < \frac{3}{2} \to f(x) = (-2x+3)\sqrt{ax} \to f'(x) = -2\sqrt{ax}$ $f'_-(\frac{3}{2}) = -\sqrt{\frac{3a}{2}} = -2\sqrt{3a}$

$f'_+(\frac{3}{2}) - f'_-(\frac{3}{2}) = 4\sqrt{3a}$ $4\sqrt{3a} = 2\sqrt{6}$ $2\sqrt{3a} = \sqrt{6}$

$12a = 6$ $a = \frac{1}{2}$ ۲

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$y = -ax + 2$ $f(2) = -2a + 2$ $f'(2) = -a$ $-2a + 2 - a = -1$

$a = \frac{3}{5}$ $f'(2) = -a = -\frac{3}{5}$ ۲

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$7y - x = 5 \to y = \frac{x+5}{7}$ $\frac{ax-1}{3x+1} = \frac{x+5}{7}$ $7ax - 7 = 3x^2 + 16x + 5$

$3x^2 + (16-7a)x + 12 = 0$ $\Delta = 0 \to 256 + 49a^2 - 224a - 144 = 0$

$(7a-4)(7a-28) = 0$ $\begin{cases} a = \frac{4}{7} \to 3x^2 + 12x + 12 = 0 \\ a = 4 \to 3x^2 - 12x + 12 = 0 \end{cases}$

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$f(2)=0 \rightarrow 8+2a-b=0 \quad f'(2)=0 \quad f'(x)=3x^2+a$

$12+a=0 \quad a=-12 \rightarrow b=-16 \quad b-a=-16-(-12)=-4$ 2

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$x=\dfrac{\sqrt{y}+1}{\sqrt{y}-1} \quad \sqrt{y}=\dfrac{x+1}{x-1} \rightarrow y=\left(\dfrac{x+1}{x-1}\right)^2 \quad y'=2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{-2}{(x-1)^2}\right)$

$f'(2)=2\times3\times(-2)=-12$ 2

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$f(-1)=-8a+8 \quad f(a)=1 \quad \dfrac{f(a)-f(-1)}{a-(-1)}=-11$

$1-(-8a+8)=-11 \quad 8a-7=-11 \quad a=-\frac{1}{2}$

$f'(x)=3(x^2+1)^2\,2x\left(-\frac{1}{2}x+1\right)+(x^2+1)^3\left(-\frac{1}{2}\right)$

$f'(1)=(3\times4\times2\times\frac{1}{2})+\left(8\times-\frac{1}{2}\right)=12-4=8$ 2

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$$\lim_{x\to 0} \frac{\sin^n(x^2)\times 2x\sin^{n-1}(x^2)\times \cos x^2}{(2\sin^2\frac{x}{2})^m} \xrightarrow{\text{هم ارزی}} \lim_{x\to 0}\frac{(x^2)^n\times n\times (x^2)^{n-1}\times 2x}{(2(\frac{x}{2})^2)^m} = \frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2m}\times \frac{1}{2}}$$

$$= \frac{2nx^{4n-1}}{\frac{1}{2^m}\times x^{2m}} = 2^{m+1}\times n\times x^{4n-2m-1} \Rightarrow 4n-2m-1=0 \rightarrow n=\frac{2m+1}{4}$$

$$\Rightarrow 2^{m+1}\times\frac{2m+1}{4} = 32\sqrt{2} \longrightarrow m=\frac{7}{2},\ n=2 \rightarrow 2m+n=9$$

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$x=\frac{\pi}{2} \rightarrow \sin\frac{\pi}{4}\cos\frac{\pi}{4}=0$

$x=\frac{\pi}{2} \rightarrow \sin\frac{\pi}{2}\cos\pi=-1$

$$\longrightarrow \frac{-1-0}{\frac{\pi}{2}} = \boxed{-\frac{4}{\pi} = \frac{\Delta y}{\Delta x}}$$

$$\sin^4 x-\cos^4 x=(\sin^2x \cancel{+} \overset{1}{} \cos^2x)(\sin^2x \cancel{-} \cos^2x) = -\cos 2x$$

($-\cos 2x$)

2

$x=\frac{\pi}{2} \rightarrow -\cos\frac{\pi}{2}=0$

$x=\frac{\pi}{2} \rightarrow -\cos\pi=1$

$$\rightarrow \frac{1-0}{\frac{\pi}{2}} = \boxed{\frac{4}{\pi} = \frac{\Delta y}{\Delta x}} \longrightarrow \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$$