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$$g\left(\frac{\pi}{4}\right)=f(1+1)=f(2)$$

$$g'(x)=\left(\frac{2\tan x}{\cos^2 x}-\sqrt{2}\sin x\right)f'(x)$$

$$g'\left(\frac{\pi}{4}\right)=3f'(2)=\sqrt{3}\longrightarrow f'(2)=\frac{\sqrt{3}}{3}$$

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$$f(x)=\frac{\cancel{(2-\cos x)}(\cos^2 x-2\cos x+4)}{(2-\cos x)\cancel{(2-\cos x)}}=\frac{\cos^2 x-2\cos x+4}{2-\cos x}$$

$$\sin x$$

$$f'\left(\frac{7\pi}{6}\right)-2g'\left(\frac{7\pi}{6}\right)=(f-2g)'\left(\frac{7\pi}{6}\right)=\left(\frac{\cos^2 x\overset{2\,-\cos x}{}-\cancel{2\cos x}}{\cancel{2-\cos x}}\right)'$$

$$\sin\frac{7\pi}{6}=-\frac{1}{2}$$

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$$x>\frac{3}{4}\longrightarrow f'(x)=4\sqrt{ax}+\frac{4ax-3a}{2\sqrt{ax}}\xrightarrow{x=\frac{3}{4}}2\sqrt{3a}$$

$$x<\frac{3}{4}\longrightarrow f'(x)=-4\sqrt{ax}+\frac{-4ax+3a}{2\sqrt{ax}}\xrightarrow{x=\frac{3}{4}}-2\sqrt{3a}$$

$$2\sqrt{3a}-(-2\sqrt{3a})=4\sqrt{3a}=2\sqrt{6}$$

$$a=\frac{1}{2}$$

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$$\left.\begin{array}{l} f'(4) = -a \\ f(4) = -4a+2 \end{array}\right\} -a-4a+2=-1 \longrightarrow a=\frac{3}{5} \longrightarrow f'(4) = -0/6$$

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$$\frac{x+5}{7} = \frac{ax-1}{3x+1} \longrightarrow 3x^2+16x+5 = 7ax-7$$

$$3x^2+(16-7a)x+12=0 \xrightarrow[\text{و مربع کامل نامنفی است}]{\text{ریشه‌ی مضاعف}} 3(x-2)^2=0$$

$$16-7a=12 \longrightarrow a=4$$

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$$\left.\begin{array}{l} f(x)' = 3x^2+a \Rightarrow f(2)' = 12+a=0 \longrightarrow a=-12^{*} \\ f(2) = 8-24-b=0 \longrightarrow b=-16^{*} \end{array}\right\} b-a = 4$$

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$$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x} = \frac{\cancel{\sin^n(x^2)}^{\,x^{2n}}}{x} = x^{2n-1}$$

$$\lim_{x\to 0} \frac{\sin^n(x^2)(x^{2n-1})}{(\cancel{1-\cos x})^m_{\;\frac{x^2}{2}}} = \frac{x^{4n-1}}{\frac{x^{2m}}{2^m}} = 2^m = 32\sqrt{2} \longrightarrow m=\frac{11}{2}$$

$$4n-1=2m=11 \longrightarrow n=3 \qquad 2m+n=14$$

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$y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \to y\sqrt{x}-y=\sqrt{x}+1 \to y\sqrt{x}-\sqrt{x}=y+1 \to \sqrt{x}(y-1)=y+1$

$\sqrt{x}=\frac{y+1}{y-1} \longrightarrow f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2$

$f^{-1\prime}(x)=2\left(\frac{x+1}{x-1}\right)\left(\frac{\overset{-2}{\cancel{(x-1)-(x+1)}}}{(x-1)^2}\right) \xrightarrow{x=2} 2\times3\times(-2)=-12$

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آهنگ متوسط در بازه $[-1,0]$: $\frac{f(0)-f(-1)}{0-(-1)}=8a-7=-11 \longrightarrow a=-\frac{1}{2}$

آهنگ لحظه‌ای در $x=1$: $f'(x)=3(x^2+1)^2\,2x\left(-\frac{x}{2}+1\right)-\frac{1}{2}(x^2+1)^3 \xrightarrow{x=1} f'(1)=8$

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① آهنگ متوسط: $\frac{\overset{1}{\cancel{(\sin\frac{\pi}{2}\cos\pi)}}-\overset{0}{\cancel{(\sin\frac{\pi}{4}\cos\frac{\pi}{2})}}}{\frac{\pi}{4}}=\frac{-4}{\pi}$

② آهنگ متوسط: $\frac{\overset{+1}{\cancel{(-\cos\pi)}}-\overset{0}{\cancel{(-\cos\frac{\pi}{2})}}}{\frac{\pi}{4}}=\frac{4}{\pi}$

$\frac{\frac{-4}{\pi}}{\frac{4}{\pi}}=-1$

$\sin^4x-\cos^4x=\underbrace{\cancel{(\sin^2x+\cos^2x)}}_{1}(\sin^2x-\cos^2x)=-\cos2x$