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1) $g'(x) = f'(\tan^2x - \sqrt{2}\cos x) \times (2\tan x(1+\tan^2x) + (-\sqrt{2}\sin x))$

$g'(\frac{\pi}{4}) = f'(2) \times 3$ $\quad 1+1 \quad 4-1$

$\rightsquigarrow f'(2)\times 3 = \sqrt{3} \longrightarrow f'(2) = \frac{\sqrt{3}}{3}$

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3) $x > \frac{3}{4}$ $\quad (4x-3)\sqrt{ax} \rightsquigarrow 4\sqrt{ax} + \frac{a}{2\sqrt{ax}} \times (4x-3) \longrightarrow \overline{8ax + 4ax - 3a}$

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4) $y + ax = 2 \rightsquigarrow y = -ax + 2$ $\quad f(4) - f'(4) = -1$

$y' = -a$ $\quad -4a + 2 - a = -1$

۲ $f'(4) = -a = -\frac{3}{5}$ $\quad 2 - 5a = -1$

$-5a = -3 \quad a = \frac{3}{5}$

5) $7y - x = 5 \rightsquigarrow y = \frac{5}{7} + \frac{x}{7}$

$f = \frac{ax-1}{3x+1} \rightsquigarrow f' = \frac{a(3x+1) - 3(ax-1)}{(3x+1)^2} = \frac{1}{7} \Rightarrow \frac{3ax + a - 3ax + 3}{(3x+1)^2} = \frac{1}{7}$

$\frac{ax-1}{3x+1} = \frac{5}{7} + \frac{x}{7} \rightsquigarrow 7ax - 7 = (5+x)(3x+1)$

$7ax - 7 = 15x + 5 + 3x^2 + x \rightarrow 3x^2 + (16-7a)x + 12$

$\Delta = (16-7a)^2 - 4\cdot 3\cdot 12 = 0 \rightsquigarrow 16 - 7a = 12 \rightarrow -7a = -4 \quad a = \frac{4}{7}$ ۲

$16 - 7a = -12 \rightarrow -7a = -28 \quad a = 4$

6) $y = x^3 + ax - b$ $\quad y' = 3x^2 + a \xrightarrow[x=2]{} 12 + a = 0 \quad a = -12$

$\xrightarrow[x=2]{} 8 - 24 - b = 0 \quad b = -16 \qquad b - a = -16 + 12 = -4$

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7) $\lim\limits_{x\to 0} \dfrac{f(x).f'(x)}{(1-\cos x)^{x}} \Longrightarrow \dfrac{f'(x).f'(x)+f''(x).f(x)}{x(1-\cos x)^{x-1}} = 42\sqrt{x}$

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8) $y = \dfrac{\sqrt{x}+1}{\sqrt{x}-1} \xrightarrow{f^{-1}} x\sqrt{y}-x=\sqrt{y}+1$

$x\sqrt{y}-\sqrt{y}=x+1$

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$\sqrt{y}(x-1)=x+1 \rightarrow y=\left(\dfrac{x+1}{x-1}\right)^{2}$

$y' = 2\left(\dfrac{x+1}{x-1}\right)\left(\dfrac{-1-1}{(x-1)^{2}}\right) \Longrightarrow 4\times -3 = -12$

13) $\dfrac{f(0)-f(-1)}{1} = -11 \qquad 1-(8)(-a+1) = 1+8a-8 \rightarrow 8a-7=-11$

$-8a+8 \qquad 8a=-4 \qquad a=-\dfrac{1}{2}$

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$x=1 \quad \left(3(x^{2}+1)^{2}\times 2x\right)\left(-\tfrac{1}{2}x+1\right)+\left(-\tfrac{1}{2}\right)(x^{2}+1)^{3} = 20$

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$$f-2g\left(\frac{7\pi}{4}\right)=\frac{(2+\cancel{\cos x})(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cancel{\cos x})}-\frac{8}{2-\cos x}=\frac{\overset{\cos x(\cancel{\cos x}-2)}{\cos^2 x-2\cos x}}{2-\cancel{\cos x}}=-\cos x$$

$$(f-2g)'=-(\cos x)'=+\sin x \longrightarrow \sin\left(\frac{7\pi}{4}\right)=\boxed{-\frac{1}{2}}$$

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$$x\to\frac{3}{4}^{+} \rightsquigarrow f(x)=4x-3\sqrt{\frac{3a}{4}} \xrightarrow[\text{صفرشونده}]{\text{مشتق عبر}} f'(x)=\frac{4\sqrt{3a}}{2}=2\sqrt{3a}$$

$$x\to\frac{3}{4}^{-} \rightsquigarrow f(x)=-4x+3\sqrt{\frac{3a}{4}} \xrightarrow[\text{صفرشونده}]{\text{مشتق عبر}} f'(x)=\frac{-4\sqrt{3a}}{2}=-2\sqrt{3a}$$

$$4\sqrt{3a}=2\sqrt{6}\to\sqrt{3a}=\frac{\sqrt{6}}{2}\to\boxed{a=\frac{1}{2}}$$

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$$\lim_{x\to 0}\frac{\sin^n(x^2)\times 2x\sin^{n-1}(x^2)\times\cos x^2}{(2\sin^2\frac{x}{2})^m}\xrightarrow{\text{هم‌ارزی}}\lim_{x\to 0}\frac{(x^2)^n\times n\times(x^2)^{n-1}\times 2x}{(2(\frac{x}{2})^2)^m}=\frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2m}\times\frac{1}{2}}$$

$$=\frac{2nx^{4n-1}}{\frac{1}{2^m}\times x^{2m}}=2^{m+1}\times n\times x^{4n-2m-1}\rightarrow 4n-2m-1=0\to n=\frac{2m+1}{4}$$

$$\hookrightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2}\longrightarrow m=\frac{7}{2},\ n=2\to\boxed{2m+n=9}$$

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$$\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-8(-a+1)}{1}=8a-7=-11\to 8a=-4\to\boxed{a=-\frac{1}{2}}$$

$$f'(x)=3(x^2+1)^2(2x)(ax+1)+a(x^2+1)^3\xrightarrow{-2a=1}f'(1)=3(2)^2(2)\left(\frac{1}{2}\right)+-\frac{1}{2}(2)^3$$

$$f'(1)=12-4=\boxed{8}$$

$x = \frac{\pi}{2} \longrightarrow \sin\frac{\pi}{4}\ \cos\frac{\pi}{2} = 0$

$x = \frac{\pi}{2} \longrightarrow \sin\frac{\pi}{2}\ \cos\pi = -1 \longrightarrow \frac{-1-0}{\frac{\pi}{2}} = \overset{1}{\boxed{-\frac{4}{\pi} = \frac{\Delta y}{\Delta x}}}$

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$\sin^4 x - \cos^4 x = (\sin^2 x \overset{1}{\cancel{+}} \cos^2 x)(\overset{-\cos 2x}{\cancel{\sin^2 x - \cos^2 x}}) = -\cos 2x$

$x = \frac{\pi}{2} \longrightarrow -\cos\frac{\pi}{2} = 0$

$x = \frac{\pi}{2} \longrightarrow -\cos\pi = 1 \longrightarrow \frac{1-0}{\frac{\pi}{2}} = \overset{2}{\boxed{\frac{4}{\pi} = \frac{\Delta y}{\Delta x}}} \longrightarrow \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$