Subject. حل‌ها Date. / /

$$g(x)=f(\tan^2 x+\sqrt{2}\cos x) \rightarrow g'(x)=\left(2(1+\tan^2 x)(\tan x)-\sqrt{2}\sin x\right) \quad ①$$

$$\sqrt{3}=3f'(2)=g'(\tfrac{\pi}{4})=(2\times2\times1-1)f'(2) \xleftarrow{g'(\frac{\pi}{4})=\sqrt{3}} \times f'(\tan^2 x+\sqrt{2}\cos x)$$

$$\hookrightarrow f'(2)=\frac{\sqrt{3}}{3}$$

$$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x} \rightarrow f'(x)=\frac{-3\sin x\cos^2 x(4-\cos^2 x)-(2\sin x\cos x)(8+\cos^3 x)}{(4-\cos^2 x)^2} \quad ②$$

$$\xrightarrow{x=\frac{7\pi}{6}} \frac{(-3\times\frac{-1}{2}\times\frac{3}{4})(4-\frac{3}{4})-(2\times\frac{-1}{2})(\frac{-\sqrt{3}}{2})(8-\frac{3\sqrt{3}}{8})}{(4-\frac{3}{4})^2}=$$

$$\frac{135-128\sqrt{3}}{169\times2}$$

$$g(x)=\frac{2}{2-\cos x} \rightarrow g'(x)=\frac{-2\sin x}{(2-\cos x)^2} \xrightarrow{x=\frac{7\pi}{6}} \frac{-2\times\frac{-1}{2}}{\frac{19+8\sqrt{3}}{4}}=$$

$$\frac{4(19-8\sqrt{3})}{169}$$

$$f'(\tfrac{7\pi}{6})-2g'(\tfrac{7\pi}{6})=\frac{135-128\sqrt{3}}{169\times2}-\frac{152-64\sqrt{3}}{169}=\frac{-169}{169\times2}=\frac{-1}{2}$$

Subject: Date: / /

(۳)

$f(x)=|۴x-۳|\sqrt{ax}$ $\quad f'_+(\frac{۳}{۴})=۴\sqrt{ax}+\frac{a}{۲\sqrt{ax}}(۴x-۳)=۴\sqrt{\frac{۳a}{۴}}$

$f'_-(\frac{۳}{۴})=-۴\sqrt{ax}+\frac{a}{۲\sqrt{ax}}(۳-۴x)=-۴\sqrt{\frac{۳a}{۴}}$

$f'_+(\frac{۳}{۴})-f'_-(\frac{۳}{۴})=۴\sqrt{\frac{۳a}{۴}}+۴\sqrt{\frac{۳a}{۴}}=\underset{\displaystyle ۲\sqrt{۱۲a}}{\underset{\downarrow}{۴\sqrt{۳a}}}=۲\sqrt{۶}\to a=\frac{۱}{۲}$

(۴)

$y+ax=۲$ $\quad f(۴)+f'(۴)=-۱\Rightarrow -۴a+۲-a=-۱\to$

$y=\frac{-۳}{۵}x+۲\to f'(۴)=\frac{-۳}{۵}$ $\quad -۵a=-۳\to a=\frac{۳}{۵}$

Subject: Date: / /

۵)

$7y - x = 5 \rightarrow y = \frac{1}{7}x + \frac{5}{7} \rightarrow y' = \frac{1}{7}$

$y = \frac{ax-1}{3x+1} \rightarrow y' = \frac{a(3x+1) - 3(ax-1)}{(3x+1)^2} \Rightarrow \frac{3ax+1-3ax+3}{(3x+1)^2} =$

$\frac{4}{(3x+1)^2} = \frac{1}{7} \Rightarrow 28 = (3x+1)^2 \rightarrow 2\sqrt{7} = 3x+1 \rightarrow \frac{2\sqrt{7}-1}{3} = x$

a هر عددی می‌تواند باشد

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۶)

$f(x) = x^3 + ax - b \qquad \rightarrow x^3 - 12x - b \rightarrow$

$(2,0) \rightarrow$ ریشه مضاعف $\rightarrow$ ریشه مشتق $\quad 8 - 24 - b = 0 \rightarrow -16 = b$

مماس شده $\qquad$ در نقطه ۲ صفر است $\qquad b - a = -16 + 12 = -4$

$3x^2 + a \rightarrow 12 + a = 0 \rightarrow a = -12$

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۷)

$f(x) = \sin^n(x^2) \rightarrow f'(x) = n(2x)(\cos(x^2))(\sin^{n-1}(x^2))$

$\lim_{x \to 0} \frac{f(x) f'(x)}{(1-\cos x)^m} = 32\sqrt{2} \rightarrow \frac{n(2x)(\cos(x^2))(\sin^{n-1}(x^2))}{(1-\cos x)^m} \rightarrow$

$\frac{n(2x)(1-\frac{x^4}{2})(x^2)^{n-1}}{(\frac{x^2}{2})^m} = 32\sqrt{2}$

$\frac{n(x)^{2n-1}}{(x)^{2m}} \times 2^{m+1} = 32\sqrt{2} \Rightarrow n \times 2^{m+1} = 32\sqrt{2}$

$2n - 1 = 2m$

Subject: Date: / /

$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow y\sqrt{x}-y=\sqrt{x}+1 \longrightarrow y\sqrt{x}-\sqrt{x}=y+1 \quad (8)$$

$$\Rightarrow \sqrt{x}(y-1)=y+1 \Rightarrow \sqrt{x}=\frac{y+1}{y-1} \rightarrow f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2$$

$$\frac{(2x+2)(x-1)^2-(2x-2)(x+1)^2}{(x-1)^4}=\frac{4(1)-(2)9}{1}=4-18=-14$$

$$\frac{f(-1)-f(0)}{-1}=\frac{8a+8-1}{-1}=-11 \rightarrow -8a+7=+11 \rightarrow \quad (9)$$

$$-8a=+4 \rightarrow a=\frac{-1}{2}$$

$$f(x)=(x^2+1)^3\left(-\tfrac{1}{2}x+1\right) \rightarrow 3(x^2+1)^2(2x)\left(-\tfrac{1}{2}x+1\right)+\left(-\tfrac{1}{2}\right)(x^2+1)^3=f'(x)$$

$$f'(1)=12-4=8$$

$$\left[\frac{\pi}{4},\frac{\pi}{2}\right] \quad y=\sin x\cos 2x \rightarrow \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1-0}{\frac{\pi}{4}}=\frac{-4}{\pi} \quad (10)$$

$$y=\sin^4 x-\cos^4 x \rightarrow \frac{g(\frac{\pi}{2})-g(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{1-0}{\frac{\pi}{4}}=\frac{4}{\pi}$$

$$\frac{\frac{-4}{\pi}}{\frac{4}{\pi}}=-1$$