۱۴ آذر

Subject: حل‌ها Date: / /

$$g(x)=f(\tan^2 x+\sqrt{2}\cos x) \rightarrow g'(x)=\left(2(1+\tan^2 x)(\tan x)-\sqrt{2}\sin x\right) \quad ①$$

$$\sqrt{3}=3f'(2)=g'\left(\tfrac{\pi}{4}\right)=(2\times2\times1-1)f'(2) \xleftarrow{g'(\frac{\pi}{4})=\sqrt{3}} \times f'(\tan^2 x+\sqrt{2}\cos x)$$

$$\hookrightarrow f'(2)=\frac{\sqrt{3}}{3}$$

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$$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x} \rightarrow f'(x)=\frac{-3\sin x\cos^2 x(4-\cos^2 x)-(2\sin x\cos x)(8+\cos^3 x)}{(4-\cos^2 x)^2} \quad ②$$

$$\xrightarrow{x=\frac{7\pi}{6}} \frac{\left(-3\times\frac{-1}{2}\times\frac{3}{4}\right)\left(4-\frac{3}{4}\right)-\left(2\times\frac{-1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)\left(8-\frac{3\sqrt{3}}{8}\right)}{\left(4-\frac{3}{4}\right)^2}=$$

$$\frac{135-128\sqrt{3}}{169\times2}$$

$$g(x)=\frac{2}{2-\cos x} \rightarrow g'(x)=\frac{-2\sin x}{(2-\cos x)^2} \xrightarrow{x=\frac{7\pi}{6}} \frac{-2\times\frac{-1}{2}}{\frac{19+8\sqrt{3}}{4}}=$$

$$\frac{4(19-8\sqrt{3})}{169}$$

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$$f'\left(\tfrac{7\pi}{6}\right)-2g'\left(\tfrac{7\pi}{6}\right)=\frac{135-128\sqrt{3}}{169\times2}-\frac{152-64\sqrt{3}}{169}=\frac{-169}{169\times2}=\frac{-1}{2}$$

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$f(x) = |۴x - ۳|\sqrt{ax} \qquad f'_+\left(\frac{۳}{۴}\right) = ۴\sqrt{ax} + \frac{a}{۲\sqrt{ax}}(۴x - ۳) = ۴\sqrt{\frac{۳a}{۴}}$ ۳

$f'_-\left(\frac{۳}{۴}\right) = -۴\sqrt{ax} + \frac{a}{۲\sqrt{ax}}(۳ - ۴x) = -۴\sqrt{\frac{۳a}{۴}}$

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$f'_+\left(\frac{۳}{۴}\right) - f'_-\left(\frac{۳}{۴}\right) = ۴\sqrt{\frac{۳a}{۴}} + ۴\sqrt{\frac{۳a}{۴}} = ۴\sqrt{۳a} = ۲\sqrt{۶} \to a = \frac{۱}{۲}$

$\downarrow$

$۲\sqrt{۱۲a}$

✓

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$y + ax = ۲ \qquad f(۴) + f'(۴) = -۱ \Rightarrow -۴a + ۲ - a = -۱ \to$ ۴

$y = \frac{-۳}{۵}x + ۲ \to f'(۴) = \frac{-۳}{۵} \qquad -۵a = -۳ \to a = \frac{۳}{۵}$

✓

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$$7y - x = 5 \longrightarrow y = \frac{1}{7}x + \frac{5}{7} \longrightarrow y' = \frac{1}{7}$$

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$$y = \frac{ax-1}{3x+1} \rightarrow y' = \frac{a(3x+1) - 3(ax-1)}{(3x+1)^2} \Rightarrow \frac{3ax+1-3ax+4}{(3x+1)^2} =$$

$$\frac{4}{(3x+1)^2} = \frac{1}{7} \Rightarrow 28 = (3x+1)^2 \longrightarrow 2\sqrt{7} = 3x+1 \longrightarrow \frac{2\sqrt{7}-1}{3} = x$$

a هر عددی می‌تواند باشد

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$$f(x) = x^3 + ax - b \qquad \longrightarrow x^3 - 12x - b \longrightarrow$$

$(2,0) \rightarrow$ محور طول‌ها مماس شده $\rightarrow$ ریشه مضاعف در نقطه ۲ صفر است $\quad 8 - 24 - b = 0 \longrightarrow -16 = b$

$$b - a = -16 + 12 = -4$$

$$3x^2 + a \longrightarrow 12 + a = 0 \longrightarrow a = -12$$

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$$f(x) = \sin^n(x^2) \longrightarrow f'(x) = n(2x)(\cos(x^2))(\sin^{n-1}(x^2))$$

$$\lim_{x\to 0} \frac{f(x)f'(x)}{(1-\cos x)^m} = 32\sqrt{2} \longrightarrow \frac{n(2x)(\cos(x^2))(\sin^{n-1}(x^2))}{(1-\cos x)^m} \rightarrow$$

$$\frac{n(2x)(1-\frac{x^4}{2})(x^2)^{n-1}}{\left(\frac{x^2}{2}\right)^m} = 32\sqrt{2}$$

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$$\frac{n(x)^{2n-1}}{(x)^{2m}} \times 2^{m+1} = 32\sqrt{2} \Rightarrow n \times 2^{m+1} = 32\sqrt{2}$$

$$2n - 1 = 2m$$

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$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow y\sqrt{x}-y=\sqrt{x}+1 \rightarrow y\sqrt{x}-\sqrt{x}=y+1 \quad ۸$$

$$\Rightarrow \sqrt{x}(y-1)=y+1 \Rightarrow \sqrt{x}=\frac{y+1}{y-1} \rightarrow f^{-1}_{(x)}=\left(\frac{x+1}{x-1}\right)^{۲}$$

ب

$$\frac{(۲x+۲)(x-1)^{۲}-(۲x-۲)(x+1)^{۲}}{(x-1)^{۲}}=\frac{۴(1)-(۲)۹}{1}=۴-۱۸=-۱۴$$

$$\frac{f_{(-1)}-f_{(0)}}{-1}=\frac{۸a+۸-1}{-1}=-11 \rightarrow -۸a+۷=+11 \rightarrow \quad ۹$$

$$-۸a=+۴ \rightarrow a=\frac{-1}{۲}$$

$$f(x)=(x^{۲}+1)^{۳}(ax+1) \rightarrow ۳(x^{۲}+1)^{۲}(۲x)(ax+1)+(a)(x^{۲}+1)^{۳}=f'(x)$$

$$f'(1)=۱۲-۴=۸$$

ب

$$\left[\frac{\pi}{۴},\frac{\pi}{۲}\right] \quad y=\sin x\cos ۲x \rightarrow \frac{f(\frac{\pi}{۲})-f(\frac{\pi}{۴})}{\frac{\pi}{۲}-\frac{\pi}{۴}}=\frac{-1-0}{\frac{\pi}{۴}}=\frac{-۴}{\pi} \quad ۱۰$$

$$y=\sin^{۴}x-\cos^{۴}x \rightarrow \frac{g(\frac{\pi}{۲})-g(\frac{\pi}{۴})}{\frac{\pi}{۲}-\frac{\pi}{۴}}=\frac{1-0}{\frac{\pi}{۴}}=\frac{۴}{\pi}$$

$$\frac{\frac{-۴}{\pi}}{\frac{۴}{\pi}}=-1$$

ب

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$$\frac{a_x - 1}{3x+1} = \frac{x}{7} + \frac{5}{7} \longrightarrow 7ax - 7 = 3x^2 + x + 15x + 5$$

$$3x^2 + x(16 - 7a) + 12 = 0 \xrightarrow{\Delta = 0} (16-7a)^2 - 12^2 = (16 - 7a - 12)(16 - 7a + 12)$$

$$a = \frac{4}{7} \qquad a = 4$$

$\boxed{a = 4} \longrightarrow 3x^2 - 12x + 12 \longrightarrow \boxed{x = 2}$ ✓

$a = \frac{4}{7} \longrightarrow 3x^2 + 12x + 12 \longrightarrow x = -2$ ناحیه اول ✗

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$$\lim_{x \to 0} \frac{\sin^n(x^2) \times 2x \sin^{n-1}(x^2) \times \cos x^2}{\left(2\sin^2 \frac{x}{2}\right)^m} \xrightarrow{\text{هم‌ارزی}} \lim_{x \to 0} \frac{(x^2)^n \times n \times (x^2)^{n-1} \times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m} = \frac{(x)^{2n+2n-2+1} \times 2n}{(x)^{2m} \times \frac{1}{2^m}}$$

$$= \frac{2nx^{4n-1}}{\frac{1}{2^m} \times x^{2m}} = 2^{m+1} \times n \times x^{4n-2m-1} \Rightarrow 4n - 2m - 1 = 0 \longrightarrow n = \frac{2m+1}{4}$$

$$\Rightarrow 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \longrightarrow m = \frac{7}{2}, \ n = 2 \longrightarrow \boxed{2m + n = 9}$$