کیمیا امیری کیا — تکلیف ۲۳

①

$$g(x) = f(\tan^2 x + \sqrt{2}\cos x) \rightarrow g'(x) = \overset{2\times2\times1}{2(1+\tan^2 x)(\tan x)} \times f'(\underset{1+1}{\tan^2 x + \sqrt{2}\cos x})$$

$$g'\left(\frac{\pi}{4}\right) = 4f'(2) = \sqrt{3} \rightarrow f'(2) = \frac{\sqrt{3}}{4}$$

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②

$$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} = \frac{\cancel{(2+\cos x)}(4-2\cos x+\cos^2 x)}{(2-\cos x)\cancel{(2+\cos x)}} = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x}$$

$$g(x) = \frac{2}{2-\cos x} \qquad f-2g = \frac{\cos^2 x - 2\cos x}{2-\cos x} = -\cos x$$

$$f' - 2g' = (f-2g)' = \sin x \xrightarrow{x=\frac{7\pi}{6}} \sin\frac{7\pi}{6} = \boxed{\frac{-1}{2}}$$

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③

$$f(x) = |4x-3|\sqrt{ax} \qquad f'(x) \begin{cases} 4\sqrt{ax} + (4x-3)\dfrac{a}{2\sqrt{ax}} & x > \frac{3}{4} \\ -4\sqrt{ax} + (-4x+3)\dfrac{a}{2\sqrt{ax}} & x < \frac{3}{4} \end{cases}$$

$$f'_+\left(\frac{3}{4}\right) = 2\sqrt{3a}$$
$$f'_-\left(\frac{3}{4}\right) = -2\sqrt{3a}$$
$$\rightarrow 2\sqrt{3a} - (-2\sqrt{3a}) = 4\sqrt{3a} = 2\sqrt{12a} = 2\sqrt{6}$$
$$\boxed{a = \frac{1}{2}}$$

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④

$$y = -ax+2 \qquad f(4) + f'(4) = -4a+2-a = -1 \rightarrow -5a = -3$$

$$a = \frac{3}{5} \qquad f'(4) = -a = \frac{-3}{5}$$

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⑤

$$y = \frac{1}{7}x + \frac{5}{7} \qquad y = \frac{ax-1}{3x+1}$$

$$y' = \frac{1}{7} \qquad y' = \frac{a+3}{3x+1}$$

$$\frac{x+5}{7} = \frac{ax-1}{3x+1} \rightarrow 7ax - 7 = 3x^2 + 16x + 5 \rightarrow 3x^2 + (16-7a) + 12 = 0$$

$\Delta = 0$ چون یک ریشه دارد $\longrightarrow x = \frac{-b}{2a} = \frac{-16+7a}{2\times3}$

$\rightarrow a = 4 \rightarrow x = 2$ ناحیه اول ✓

$\rightarrow a = \frac{4}{7} \rightarrow x = -2$ ناحیه دوم ✗

(۶)

$$\left.\begin{array}{l} f(x)=x^3+ax-b \\ f(2)=0 \end{array}\right\} \to \left.\begin{array}{l} x^3+ax-b=0 \\ x=2 \end{array}\right\} \to 8+2a-b=0 \to 2a-b=-8$$

$$f'(x)=3x^2+a \xrightarrow{x=2} 12+a=0 \to a=-12$$

$$-24-b=-8$$

$$b=8-24=-16$$

$$b-a=-16-(-12)=-4$$

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(۷)

$$f(x)=\sin^n(x^2) \qquad \lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m}=32\sqrt{2} \qquad 2m+n?$$

$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\frac{\sin^n(x^2)}{x}=x^{2n-1}$$

$$32\sqrt{2}=\frac{\underbrace{\sin^n(x^2)}_{x^{2n}}(x^{2n-1})}{\underbrace{(1-\cos x)^m}_{\frac{x^2}{2}}}=\frac{\cancel{x^{4n-1}}}{\frac{1}{2^m}\times \cancel{x^{2m}}}=2^m=32\sqrt{2}$$

$$2^m=32\sqrt{2}=2^{\frac{11}{2}} \to m=\frac{11}{2} \qquad 2m=11=4n-1 \to n=3$$

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(۸)

$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \to f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2$$

$$(f^{-1})'=2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right) \to (f^{-1})'(2)=2\times 3\times(-2)=\boxed{-12}$$

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(۹)

$$f(x)=(x^2+1)^3(ax+1) \quad \xrightarrow{x=0} =1 \qquad \xrightarrow{x=-1} =-8a+8$$

$$\frac{1+8a-8}{0-(-1)}=+8a-7=-11$$

$$8a=-4 \to a=\frac{-1}{2}$$

$$-2a=1 \qquad f'(x)=3(x^2+1)^2\times 2x\left(\frac{-1}{2}x+1\right)+(x^2+1)^3\left(\frac{-1}{2}\right)$$

$$f'(1)=6\times 4\times\frac{1}{2}+8\times\left(\frac{-1}{2}\right)=12-4=8$$

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(۱۰)

$$y=\sin x\cos 2x \qquad y=\sin^4 x-\cos^4 x=\sin^2 x-\cos^2 x=-\cos 2x$$

$$x=\frac{\pi}{2} \to y=-1 \qquad x=\frac{\pi}{2} \to y=1$$

$$x=\frac{\pi}{4} \to y=0 \qquad x=\frac{\pi}{4} \to y=0$$

$$\frac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1}{\frac{\pi}{4}}=\frac{-4}{\pi} \qquad \frac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{4}{\pi} \qquad \frac{-4\pi}{4\pi}=-1$$