لیمیا امیری کیا   تکلیف ۲۳   ۱۹/۵ آفرین

①
$$g(x)=f(\tan^2 x+\sqrt{2}\cos x)\rightarrow g'(x)=2(1+\tan^2 x)(\tan x)\times f'(\tan^2 x+\sqrt{2}\cos x)$$
$$g'(\tfrac{\pi}{4})=4f'(2)=\sqrt{3}\rightarrow f'(2)=\frac{\sqrt{3}}{4}$$

۱/۵

②
$$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}=\frac{\cancel{(2+\cos x)}(4-2\cos x+\cos^2 x)}{(2-\cos x)\cancel{(2+\cos x)}}=\frac{\cos^2-2\cos x+4}{2-\cos x}$$
$$g(x)=\frac{2}{2-\cos x}\qquad f-2g=\frac{\cos^2-2\cos x}{2-\cos x}=-\cos x$$
$$f'-2g'=(f-2g)'=\sin x\xrightarrow{x=\frac{7\pi}{6}}\sin\frac{7\pi}{6}=\boxed{\frac{-1}{2}}$$

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③
$$f(x)=|4x-3|\sqrt{ax}\qquad f'(x)\begin{cases}4\sqrt{ax}+(4x-3)\frac{a}{2\sqrt{ax}} & x>\frac{3}{4}\\ -4\sqrt{ax}+(-4x+3)\frac{a}{2\sqrt{ax}} & x<\frac{3}{4}\end{cases}$$
$$f'_+(\tfrac{3}{4})=2\sqrt{3a}$$
$$f'_-(\tfrac{3}{4})=-2\sqrt{3a}$$
$$\rightarrow 2\sqrt{3a}-(-2\sqrt{3a})=4\sqrt{3a}=2\sqrt{12a}=2\sqrt{6}$$
$$\boxed{a=\frac{1}{2}}$$

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④
$$y=-ax+2\qquad f(4)+f'(4)=-4a+2-a=-1\rightarrow -5a=-3$$
$$a=\frac{3}{5}\qquad f'(4)=-a=\frac{-3}{5}$$

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⑤
$$y=\frac{1}{7}x+\frac{5}{7}\qquad y=\frac{ax-1}{3x+1}$$
$$y'=\frac{1}{7}\qquad y'=\frac{a+3}{3x+1}$$

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$$\frac{x+5}{7}=\frac{ax-1}{3x+1}\rightarrow 7ax-7=3x^2+16x+5\rightarrow 3x^2+(16-7a)+12=0$$

$\Delta=0$
چون یک ریشه دارد

$$\rightarrow x=\frac{-b}{2a}=\frac{-16+7a}{2\times 3}$$

$a=4\rightarrow x=2$ ناحیه اول ✓

$a=\frac{4}{7}\rightarrow x=-2$ ناحیه دوم ✗

(6)

$$f(x)=x^3+ax-b,\quad f(x)=0 \;\Big\}\to x^3+ax-b=0,\ x=2 \;\Big\}\to 8+2a-b=0 \to 2a-b=-8$$

$$f'(x)=3x^2+a \xrightarrow{x=2} 12+a=0 \to a=-12$$

$$-24-b=-8$$

$$b=8-24=-16$$

$$b-a=-16-(-12)=-4$$

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(7)

$$f(x)=\sin^n(x^2) \qquad \lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m}=32\sqrt{2} \qquad 2m+n?$$

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$$f'(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}=\frac{\sin^n(x^2)}{x}=x^{2n-1}$$

$$32\sqrt{2}=\frac{\underbrace{\sin^n(x^2)}_{x^{2n}}(x^{2n-1})}{\underbrace{(1-\cos x)^m}_{\frac{x^2}{2}}}=\frac{\cancel{x^{4n-1}}}{\frac{1}{2^m}\times \cancel{x^{2m}}}=2^m=32\sqrt{2}$$

$$2^m=32\sqrt{2}=2^{\frac{11}{2}} \to m=\frac{11}{2} \qquad 2m=11=4n-1 \to n=3$$

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(8)

$$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \longrightarrow f^{-1}(x)=\left(\frac{x+1}{x-1}\right)^2$$

$$(f^{-1})'=2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right) \to (f^{-1})'(3)=2\times 3\times(-2)=\boxed{-12}$$

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(9)

$$f(x)=(x^2+1)^3(ax+1) \quad \xrightarrow{x=0}\ =1 \qquad \xrightarrow{x=-1}\ =-8a+8$$

$$\frac{1+8a-8}{0-(-1)}=+8a-7=-11$$

$$8a=-4 \to a=\frac{-1}{2}$$

$$-2a=1$$

$$f'(x)=3(x^2+1)^2\times 2x\left(\frac{-1}{2}x+1\right)+(x^2+1)^3\left(\frac{-1}{2}\right)$$

$$f'(1)=6\times 4\times\frac{1}{2}+8\times\left(\frac{-1}{2}\right)=12-4=8$$

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(10)

$$y=\sin x\cos 2x \qquad y=\sin^4 x-\cos^4 x=\sin^2 x-\cos^2 x=-\cos 2x$$

$$x=\frac{\pi}{2}\to y=-1 \qquad x=\frac{\pi}{2}\to y=1$$

$$x=\frac{\pi}{4}\to y=0 \qquad x=\frac{\pi}{4}\to y=0$$

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$$\frac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1}{\frac{\pi}{4}}=\frac{-4}{\pi} \qquad \frac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{4}{\pi} \qquad \frac{-4\pi}{4\pi}=-1$$

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$$g'(x) = f'(\tan^{۲}x + \sqrt{۲}\cos x) \times (۲\tan x(۱+\tan^{۲}x) - \sqrt{۲}\sin x)$$ ۱

$$g'(\frac{\pi}{۴}) = f'(۱ + \sqrt{۲} \times \frac{\sqrt{۲}}{۲}) \times ۲ \times ۱ \times (۱+۱^{۲}) - (\sqrt{۲} \times \frac{\sqrt{۲}}{۲})$$

$$\underset{\sqrt{۳}}{\cancel{g'(\frac{\pi}{۴})}} = f'(۲) \times [\underset{۳}{\cancel{۲(۲)-۱}}] \longrightarrow f'(۲) = \frac{\sqrt{۳}}{۳}$$

$$\lim_{x\to ۰} \frac{\sin^{n}(x^{۲}) \times ۲x\sin^{n-۱}(x^{۲}) \times \cos x^{۲}}{(۲\sin^{۲}\frac{x}{۲})^{m}} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to ۰} \frac{(x^{۲})^{n} \times n \times (x^{۲})^{n-۱} \times ۲x}{(۲(\frac{x}{۲})^{۲})^{m}} = \frac{(x)^{۲n+۲n-۲+۱} \times ۲n}{(x)^{۲m} \times \frac{۱}{۲}}$$ ۷

$$= \frac{۲nx^{۴n-۱}}{\frac{۱}{۲^{m}} \times x^{۲m}} = ۲^{m+۱} \times n \times x^{۴n-۲m-۱} \to ۴n-۲m-۱=۰ \to n = \frac{۲m+۱}{۴}$$

$$\to ۲^{m+۱} \times \frac{۲m+۱}{۴} = ۳۲\sqrt{۲} \longrightarrow m = \frac{۷}{۲},\ n = ۲ \to \boxed{۲m+n=۹}$$