Subject: هلیا ابوطالبی
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$g'(x) = 2\tan x(1+\tan^2 x) - \sqrt{2}\sin x\, f'(x)$

$g'(x) = 2\tan\frac{\pi}{4}\left(1+\tan^2\frac{\pi}{4}\right) - \sqrt{2}\times\frac{\sqrt{2}}{2}\times f'(2)$

$\sqrt{3} = 4 - f'(2) \qquad f'(2) = 4-\sqrt{3}$

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$f(x) = \dfrac{8+\cos^3 x}{4-\cos^2 x} = \dfrac{4-2\cos x+\cos^2 x}{2-\cos x}$

$\dfrac{4-2\cos x+\cos^2 x}{2-\cos x} - 2\left(\dfrac{2}{2-\cos x}\right) = \dfrac{-2\cos x+\cos^2 x}{2-\cos x} = \dfrac{-\cos x\,\cancel{(2-\cos x)}}{\cancel{2-\cos x}} = -\cos x$

$f - 2g = -\cos x \Rightarrow f' - 2g' = +\sin x \qquad \sin\frac{7\pi}{6} = -\frac{1}{2}$

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$\frac{3}{4}^{+} \to (4x-3)(\sqrt{ax}) \qquad \frac{3}{4}^{-} \Rightarrow -(4x-3)\sqrt{ax}$

علامت عوض نمی‌شود　　　علامت عوض نمی‌کند

$(4x-3)'\sqrt{ax} \to 4\sqrt{ax} \qquad = -4\sqrt{ax}$

$4\sqrt{ax} - (-4\sqrt{ax}) = 8\sqrt{ax} = \cancel{8}\sqrt{a\,\tfrac{3}{4}} = \cancel{2}\sqrt{6}$

$2\sqrt{3a} = \sqrt{6} \Rightarrow a = \frac{\sqrt{2}}{2}$

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$y = -ax+2 \qquad f(4)+f'(4) = -1$

$-4a+2-a = -1$

$-5a+2 = -1 \qquad -5a = -3$

$\Rightarrow y = -\frac{3}{5}x+2 \qquad a = \frac{3}{5}$

$f'(x) = \frac{-3}{5}$

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$$y = \frac{x+5}{7}$$

$$\frac{ax-1}{3x+1} = \frac{x+5}{7} \Rightarrow \frac{7ax - 7 - (3x^2 + 16x + 5)}{(3x+1)^2}$$

$$7ax - 7 - 3x^2 - 16x - 5 \Rightarrow$$

$$-3x^2 - 16x + 7ax - 12 \Rightarrow \quad \Delta = 0 \Rightarrow$$

$$b^2 - 4ac \Rightarrow \quad b^2 - 4 \times -3 \times -12 = 0$$

$$b^2 = +144 \Rightarrow b = \pm 12$$

$b = +12 \Rightarrow a = 4 \Rightarrow x = -2$ غق

$$\text{if } b = -12 \quad a = \frac{4}{7} \Rightarrow \frac{a-3}{(3x+1)^2} = -\left(\frac{4}{7} - 3\right) = \frac{-17/7}{(3x+1)^2}$$

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$$f'(x) = 3x^2 + a \Rightarrow$$

$$f'(2) = 12 + a = 0 \Rightarrow \quad a = -12$$

$$f(2) = 0 \Rightarrow 8 - 24 - b = 0 \Rightarrow \quad b = -16$$

$$b - a = -16 - (-12) = -4$$

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$\sin^n(x^2) = x^{2n}$

$$\frac{x^{2n} \cdot 2n\,(x^{2n}-1)}{\left(1-\left(1-\frac{x^2}{2}\right)\right)^m} = \frac{2n\, x^{2n} \times (x^{2n}-1)}{\left(\frac{x^2}{2}\right)^m} \Rightarrow$$

$$\frac{2^m \times \cancel{2n} \times x^{4n-1}}{x^{2m}} = \cancel{16}\sqrt{2}$$

$2m = 4n-1 \qquad 4n = 2m+1 \qquad n = \frac{2m+1}{4}$

$2^m \times \frac{2m+1}{4} = 16\sqrt{2} = 2^{\frac{m}{2}}(2m+1) = 64\sqrt{2}$ if $m=\frac{7}{2}$ ✓

$2m = 7$; $7 = 4n-1 \Rightarrow n=2$ $2m+n = 7+2 = 9$

$$f(x) = \frac{(\sqrt{x}+1)}{\sqrt{x}-1} = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - \frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2}$$

$$= \frac{\frac{1}{2\sqrt{2}}\left(\sqrt{2}-1-\sqrt{2}-1\right)}{(\sqrt{2}-1)^2} = \frac{-\frac{2\times 1}{2\sqrt{2}}}{(\sqrt{2}-1)^2} = \frac{-1}{\sqrt{2}(\sqrt{2}-1)^2}$$

$-\sqrt{2}(\sqrt{2}-1)^2$

$-1 \Rightarrow (1+1)^3(-a+1) = 8(1-a)$ -9

$0 \rightarrow 1(1) = 1$ $8-8a$

$1 - 8 + 8a = -11 \Rightarrow 8a = -4 \Rightarrow a = -\frac{1}{2}$

$$\frac{f(0)-f(-1)}{0-(-1)} = -11 \Rightarrow f(x) = (x^2+1)^3\left(-\frac{1}{2}x+1\right)$$

$$f'(x) = 3\times 2x \times (x^2+1)^2\left(-\frac{1}{2}x+1\right) + \left(-\frac{1}{2}\right)(x^2+1)^3$$

$$3\times 2\times 2\times 4 \times \frac{1}{\cancel{4}} = 12 + \frac{-1}{\cancel{2}} \times 8 = 12 - 4 = 8$$

$-b$

$$\frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{\sin\frac{\pi}{2}\cos\pi - \sin\frac{\pi}{4}\cos\frac{\pi}{2}}{\frac{\pi}{4}}$$

$$= \frac{-1}{\frac{\pi}{4}} = \frac{-4}{\pi}$$

$$g(x) = \cancel{(\sin^2 a + \cos^2 a)}(\sin^2 a - \cos^2 a) = s$$

$$\frac{g(\frac{\pi}{2}) - g(\frac{\pi}{4})}{\frac{\pi}{4}} = \frac{1-0}{\frac{\pi}{4}} = \frac{4}{\pi} = s$$

$$\frac{\frac{-4}{\pi}}{\frac{4}{\pi}} = -1$$