Subject: هلیا ابوطالبی   ۱۸/۵

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۱ـ

$g'(x) = 2\tan x(1+\tan x) - \sqrt{2}\sin x\, f'(x)$

$g'(x) = 2\cancel{\tan\frac{\pi}{4}}\left(1+\cancel{\tan\frac{\pi}{4}}\right) - \sqrt{2}\times\frac{\sqrt{2}}{2}\times f'(2)$ ۲/۵

$\sqrt{3} = 4 - f'(2)$ $f'(2) = 4-\sqrt{3}$

۲ـ

$f(x) = \frac{8+\cos^3 x}{4-\cos^2 x} = \frac{4-2\cos x+\cos^2 x}{2-\cos x}$

$\frac{4-2\cos x+\cos^2 x}{2-\cos x} - 2\left(\frac{2}{2-\cos x}\right) = \frac{-2\cos x+\cos^2 x}{\cancel{2-\cos x}} = -\cos x$ $-\cos x\,\cancel{(2-\cos x)}$

$f - 2g = -\cos x \Rightarrow f' - 2g' = +\sin x$ ۲ $\sin\frac{7\pi}{6} = -\frac{1}{2}$

۳ـ

$\frac{3}{4}^+ \to (4x-3)(\sqrt{ax})$ $\frac{3}{4}^- \Rightarrow -(4x-3)\sqrt{ax}$

علامت عوض نمیشه   ۱/۷۵

$(4x-3)'\sqrt{ax} \to 4\sqrt{ax}$ $= -4\sqrt{ax}$

$4\sqrt{ax} - (-4\sqrt{ax}) =$ $\cancel{8}\sqrt{a\frac{3}{4}} = \cancel{2}\sqrt{6}$

$2\sqrt{\cancel{3}a} = \sqrt{6} = a = \frac{\sqrt{2}}{2}$

۴ـ

$y = -ax+2$ $f(4)+f'(4) = -1$

$-4a+2-a=-1$

$-5a+2=-1$ $-5a=-3$

$\Rightarrow y = -\frac{3}{5}x+2$ $a=\frac{3}{5}$

$f'(x) = -\frac{3}{5}$ ۲

ـ۵ $y = \frac{x+5}{7}$

$\frac{ax-1}{3x+1} = \frac{x+5}{7} \Rightarrow \frac{7ax-7-(3x^2+16x+5)}{(3x+1)^2}$

$7ax - 7 - 3x^2 - 16x - 5 \Rightarrow$

$-3x^2 - 16x + 7ax - 12 \Rightarrow \quad \Delta = 0 \Rightarrow$ ۱/۵

$b^2 - 4ac \Rightarrow \quad b^2 - 4 \times -3 \times -12 = 0$

$b^2 = 144 \Rightarrow b = \pm 12$

$b = 12 \Rightarrow a = 4 \Rightarrow x = 2$ غق

if $b = -12 \quad a = \frac{4}{7} \Rightarrow \frac{a-3}{(3x+1)^2} = -\frac{\frac{4}{7}-3}{(3x+1)^2} = \frac{-17}{7}$

قق

ـ۶ $f'(x) = 3x^2 + a \Rightarrow$

$f'(2) = 12 + a = 0 \Rightarrow \quad a = -12$

$f(2) = 0 \Rightarrow 8 - 24 - b = 0 \Rightarrow b = -16$

$b - a = -16 - (-12) = -4$

۲

$\sin^n(x^2) = x^{2n}$

$$\frac{k^{2n}\cdot 2n\,(x^{2n}-1)}{\left(1-\left(1-\frac{x^2}{2}\right)\right)^m} = \frac{2n\, x^{2n}\times(x^{2n}-1)}{\left(\frac{x^2}{2}\right)^m} \Rightarrow$$

$$2^m \times \frac{2n \times x^{4n-1}}{x^{2m}} = 16\sqrt{2}$$

$2m = 4n-1 \qquad 4n = 2m+1 \qquad n = \frac{2m+1}{4}$

$$\times 2^m \times \frac{2m+1}{4} = 16\sqrt{2} = 2^{\frac{m}{2}}(2m+1) = 64\sqrt{2} \qquad \text{if } m=\frac{7}{2}$$

$2m = 7 \quad 7 = 4n-1 \Rightarrow n = 2 \quad 2m+n = 7+2 = 9$

$$f(x) = \frac{(\sqrt{x}+1)}{\sqrt{x}-1} = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - \frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2}$$

$$= \frac{\frac{1}{2\sqrt{2}}\left(\sqrt{2}-1-\sqrt{2}-1\right)}{(\sqrt{2}-1)^2} = \frac{\frac{-2\times 1}{2\sqrt{2}}}{(\sqrt{2}-1)^2} = \frac{-1}{\sqrt{2}(\sqrt{2}-1)^2}$$

$-\sqrt{2}(\sqrt{2}-1)^2$

15

$-1 \Rightarrow (+1+1)^3(-a+1) = 8(1-a)$ -9

$0 \rightarrow 1(1) = 1 \qquad 8-8a$

$1-8+8a = -11 \Rightarrow 8a = -4 \Rightarrow a = -\frac{1}{2}$

$$\frac{f(0)-f(-1)}{0-(-1)} = -11 \Rightarrow f(x) = (x^2+1)^3\left(-\frac{1}{2}x+1\right)$$

$$f'(x) = 3\times 2x\,(x^2+1)^2\left(-\frac{1}{2}x+1\right) + \left(-\frac{1}{2}\right)(x^2+1)^3$$

$$3\times 2\times 2\times 4\times\frac{1}{\cancel{2}} = 12 + \frac{-1}{\cancel{2}}\times 8 = 12-4 = 8$$

2

b

$$\frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{\sin\frac{\pi}{2}\cos\pi - \sin\frac{\pi}{4}\cos\frac{\pi}{2}}{\frac{\pi}{4}}$$

$$= \frac{-1}{\frac{\pi}{4}} = \frac{-4}{\pi}$$

$$g(u) = \underbrace{\cancel{(\sin^2 a + \cos^2 a)}}_{1}(\sin^2 a - \cos^2 a)$$

$$\frac{g(\frac{\pi}{2}) - g(\frac{\pi}{4})}{\frac{\pi}{4}} = \frac{1 - 0}{\frac{\pi}{4}} = \frac{4}{\pi}$$

$$\frac{\frac{-4}{\pi}}{\frac{4}{\pi}} = -1$$

۵

$$g(x) = f'(\tan^۲ x + \sqrt{۲}\cos x) \times (۲\tan x(۱+\tan^۲ x) - \sqrt{۲}\sin x)$$ ۱

$$g'(\frac{\pi}{۴}) = f'(۱ + \sqrt{۲}\times\frac{\sqrt{۲}}{۲}) \times ۲\times۱\times(۱+۱^۲) - (\sqrt{۲}\times\frac{\sqrt{۲}}{۲})$$

$$\underbrace{\cancel{g'(\frac{\pi}{۴})}}_{\sqrt{۳}} = f'(۲) \times [\underbrace{\cancel{۲(۲)}-۱}_{۳}] \longrightarrow f'(۲) = \frac{\sqrt{۳}}{۳}$$

$$x \to \frac{۳}{۴}^+ \leadsto f(x) = ۴x - ۳\sqrt{\frac{۳a}{۴}} \xrightarrow[\text{صفرشونده}]{\text{مشتق عبر}} f'(x) = \frac{۴\sqrt{۳a}}{۲} = ۲\sqrt{۳a}$$ ۳

$$x \to \frac{۳}{۴}^- \leadsto f(x) = -۴x + ۳\sqrt{\frac{۳a}{۴}} \xrightarrow[\text{صفرشونده}]{\text{مشتق عبر}} f'(x) = \frac{-۴\sqrt{۳a}}{۲} = -۲\sqrt{۳a}$$

$$۴\sqrt{۳a} = ۲\sqrt{۶} \to \sqrt{۳a} = \frac{\sqrt{۶}}{۲} \to \boxed{a = \frac{۱}{۲}}$$

$$\frac{ax-۱}{۳x+۱} = \frac{x}{۷} + \frac{۵}{۷} \longrightarrow ۷ax - ۷ = ۳x^۲ + x + ۱۵x + ۵$$ ۵

$$۳x^۲ + x(۱۶-۷a) + ۱۲ = ۰ \xrightarrow{\Delta=۰} (۱۶-۷a)^۲ - ۱۲^۲ = \underbrace{(۱۶-۷a-۱۲)}_{\boxed{a=\frac{۴}{۷}}}\underbrace{(۱۶-۷a+۱۲)}_{\boxed{a=۴}}$$

$$\boxed{a=۴} \longrightarrow ۳x^۲ - ۱۲x + ۱۲ \longrightarrow \boxed{x=۲} \checkmark$$

$a = \frac{۴}{۷} \longrightarrow ۳x^۲ + ۱۲x + ۱۲ \longrightarrow x = -۲$ × ناحیه اول

$$f^{-۱}(۲) = t \to f(t) = ۲ \longrightarrow \sqrt{t} + ۱ = ۲\sqrt{t} - ۲ \longrightarrow \sqrt{t} = ۳ \longrightarrow t = ۹$$ ۸

$${f^{-۱}}'(۲) = \frac{۱}{f'(۹)} \quad , \quad f'(x) = \frac{-۲}{(\sqrt{x}-۱)^۲} \times \frac{۱}{۲\sqrt{x}}$$

$$f'(۹) = \frac{-۲}{(۲)^۲} \times \frac{۱}{۶} = -\frac{۱}{۱۲} \longrightarrow \boxed{{f^{-۱}}'(۲) = -۱۲}$$