صبا خازاده   تکلیف شماره ۲۳   دوازدهم دختر B

۱)
$$g'(x) = \big((2\tan x)(1+\tan^2 x) - \sqrt{2}\sin x\big) f'(\tan^2 x + \sqrt{2}\cos x) \qquad g'(\tfrac{\pi}{4}) = (2\times 2-1) f'(1+1)$$
$$\sqrt{3} = 3f'(2) \qquad f'(2) = \frac{\sqrt{3}}{3}$$

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۲)
$$f(x) - 2g(x) = \frac{8+\cos^3 x}{4-\cos^2 x} - \frac{4}{2-\cos x} = \frac{8+\cos^3 x - 8 - 4\cos x}{4-\cos^2 x} = \frac{\cos x(\cos^2 x - 4)}{4-\cos^2 x} = -\cos x$$
$$f'(x) - 2g'(x) = \sin x \qquad f'(\tfrac{7\pi}{6}) - 2g'(\tfrac{7\pi}{6}) = \sin\frac{7\pi}{6} = -\frac{1}{2}$$

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۳)
$$x > \frac{3}{4} \to f(x) = (4x-3)\sqrt{ax} \to f'(x) = 4\sqrt{ax} \qquad f'_+(\tfrac{3}{4}) = 4\sqrt{\frac{3a}{4}} = 2\sqrt{3a}$$

$x = \frac{3}{4}$ عامل صفرشونده است.

$$x < \frac{3}{4} \to f(x) = (-4x+3)\sqrt{ax} \to f'(x) = -4\sqrt{ax} \qquad f'_-(\tfrac{3}{4}) = -4\sqrt{\frac{3a}{4}} = -2\sqrt{3a}$$
$$f'_+(\tfrac{3}{4}) - f'_-(\tfrac{3}{4}) = 4\sqrt{3a} \qquad 4\sqrt{3a} = 2\sqrt{6} \qquad 2\sqrt{3a} = \sqrt{6} \to 12a = 6 \qquad a = \frac{1}{2}$$

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۴)
$$y = -ax+2 \qquad f(4) = -4a+2 \qquad f'(4) = -a \qquad -4a+2-a = -1 \qquad a = \frac{3}{5} \qquad f'(4) = -a = -\frac{3}{5}$$

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۵)
$$7y - x = 5 \to y = \frac{x+5}{7} \qquad \frac{ax-1}{3x+1} = \frac{x+5}{7} \qquad 7ax - 7 = 3x^2 + x + 15x + 5$$
$$3x^2 + (16-7a)x + 12 = 0 \qquad \Delta = 0 \to 256 + 49a^2 - 224a - 144 = 0 \qquad 49a^2 - 224a - 112 = 0$$
$$(7a-4)(7a-28) = 0 \qquad \begin{cases} a = \frac{4}{7} \to 3x^2 + 12x + 12 = 0 \quad 3(x+2)^2 = 0 \quad x = -2 \ \times \\ a = 4 \to 3x^2 - 12x + 12 = 0 \quad 3(x-2)^2 = 0 \quad x = 2 \ \checkmark \end{cases}$$

درناحیه اول نیست

پس $\boxed{a = 4}$

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۶)
$$f(2) = 0 \to 8 + 2a - b = 0 \qquad f'(2) = 0 \qquad f'(x) = 3x^2 + a \qquad 12 + a = 0 \qquad a = -12 \to b = -16$$
$$b - a = -16 - (-12) = -4$$

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۷)
$$f'(x) = n\sin^{n-1}(x^2)\cdot\cos(x^2)\cdot 2x \qquad \lim_{x\to 0} \frac{\sin^n(x^2)\times n\sin^{n-1}(x^2)\times\cos(x^2)\times 2x}{(1-\cos x)^m}$$
$$\xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0} \frac{x^{2n}\times n x^{2n-2}\times 1\times 2x}{(\frac{x^2}{2})^m} = \lim_{x\to 0} \frac{2n\times x^{4n-1}\times 2^m}{x^{2m}} = 32\sqrt{2} = 2^{\frac{11}{2}}$$

باید $4n - 1 = 2m$ باشد.

$$n\times 2^{m+1} = 2^{\frac{11}{2}} \qquad n = 2 \to m = \frac{7}{2} \qquad 2m + n = 2(\tfrac{7}{2}) + 2 = 9$$

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۸)
$$x = \frac{\sqrt{y}+1}{\sqrt{y}-1} \qquad \sqrt{y} = \frac{x+1}{x-1} \to y = \left(\frac{x+1}{x-1}\right)^2 \qquad y' = 2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right) \qquad f'(2) = 2\times 3\times(-2) = -12$$

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۹)
$$f(-1) = -8a + 8 \qquad f(0) = 1 \qquad \frac{f(0) - f(-1)}{0-(-1)} = -11 \qquad 1 - (-8a+8) = -11 \qquad 8a - 7 = -11 \qquad a = -\frac{1}{2}$$
$$f'(x) = 3(x^2+1)^2(2x)(-\tfrac{1}{2}x + 1) + (x^2+1)^3(-\tfrac{1}{2}) \qquad f'(1) = (3\times 4\times 2\times\tfrac{1}{2}) + (8\times -\tfrac{1}{2}) = 12 - 4 = 8$$

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۱۰)
$$f(\tfrac{\pi}{4}) = 0 \qquad f(\tfrac{\pi}{2}) = -1 \qquad \frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{-1-0}{\frac{\pi}{4}} \qquad y = \sin^2 x - \cos^2 x = -\cos 2x$$
$$g(\tfrac{\pi}{4}) = 0 \qquad g(\tfrac{\pi}{2}) = 1 \qquad \frac{g(\frac{\pi}{2}) - g(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{1-0}{\frac{\pi}{4}} \qquad \frac{\frac{-1}{\frac{\pi}{4}}}{\frac{1}{\frac{\pi}{4}}} = -1$$