نام: دینا احمدی   شمارهٔ تکلیف: ۲۳   مقطع: دختران دوازدهم B

① $g'(x)=\left(2\tan x(1+\tan^2 x)-\sqrt{2}\sin x\right).f'(\tan^2 x+\sqrt{2}\cos x)\to \tan^2 x+\sqrt{2}\cos x=2$

$\to x=\frac{\pi}{4}\to g'(\frac{\pi}{4})=\sqrt{3}=\left(2\tan(\frac{\pi}{4})(1+\tan^2(\frac{\pi}{4}))-\sqrt{2}\sin(\frac{\pi}{4})\right)\times f'(2)\to f'(2)=\boxed{\frac{\sqrt{3}}{3}}$

② $\cos(\frac{7\pi}{6})=-\frac{\sqrt{3}}{2}\ ,\ \sin(\frac{7\pi}{6})=-\frac{1}{2}\Rightarrow g'(x)=\frac{2\sin x}{(2-\cos^2 x)^2}\Rightarrow g'(\frac{7\pi}{6})=\frac{76-32\sqrt{3}}{169}$

$f'(x)=\frac{(-3\cos^2 x\sin x)(4-\cos^2 x)-(8+\cos^3 x)(-2\sin x\cos x)}{(4-\cos^2 x)^2}\Rightarrow f'(\frac{7\pi}{6})=\frac{135-128\sqrt{3}}{338}$

$\longrightarrow f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=\frac{135-128\sqrt{3}}{338}-\frac{304-128\sqrt{3}}{338}=\boxed{\frac{-1}{2}}$

③ $f'_+(x)=4\times\sqrt{ax}+(4x-3)\frac{a}{2\sqrt{ax}}\quad,\quad f'_-(x)=-4\times\sqrt{ax}+-(4x-3)\frac{a}{2\sqrt{ax}}$

$f'_+(\frac{3}{4})-f'_-(\frac{3}{4})=2\sqrt{6}\to 8\sqrt{ax}+(8x-6)\frac{a}{2\sqrt{ax}}=2\sqrt{6}\to 4\sqrt{3a}=2\sqrt{6}\to a=\boxed{\frac{1}{2}}$

④ $y=-ax+2\to f(4)=-4a+2\ ,\ f'(4)=-a\to f(4)+f'(4)=-1\to -5a=3\to a=\frac{3}{5}$

$f'(4)=-a=\boxed{\frac{-3}{5}}$

⑤ $y=\frac{1}{7}x+\frac{5}{7}\to$ شیب $m=\frac{1}{7}\to y=\frac{ax-1}{3x+1}\to y'=\frac{a+3}{(3x+1)^2}=\frac{1}{7}\to 9x^2+6x-20-7a=0$ ①

$\frac{ax-1}{3x+1}=\frac{x+5}{7}\to 3x^2+(16-7a)x+12=0\xrightarrow{\times 3}9x^2+3(16-7a)x+36=0$ ②

$\xrightarrow{①,②}x=1\to 9+6-20=7a\to a=\boxed{\frac{-5}{7}}$

⑥ بر محور x ها مماس $\to f'(2)=0\to 3x^2+a=0\to a=-12$

$\hookrightarrow f(2)=0\to 8+2(-12)=b\to b=-16\ \Big\}\to b-a=\boxed{-4}$

⑦ $\sin(x^2)\sim x^2\to f(x)=\sin^n(x^2)\sim(x^2)^n=x^{2n}$

$f(x)=\sin^n(x^2)\to f'(x)=n\sin^{n-1}(x^2).2x.\cos(x^2)\to f'(x)\sim 2nx^{2n-2}.x=2nx^{2n-1}$

صورت $\Rightarrow f'(x).f(x)\sim 2nx^{4n-1}$

مخرج $\Rightarrow (1-\cos x)^m\sim\left(\frac{x^2}{2}\right)^m$ $\Big\}\to \lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m}=2n2^m x^{4n-1-2m}$

اگر توان صفر نشود حد صفر یا بی‌نهایت $\xrightarrow{\text{حد موجود است}}$ $4n-1-2m=0\to m=2n-\frac{1}{2}$

$2n2^m=32\sqrt{2}\to 2n2^{2n-\frac{1}{2}}=2^{\frac{11}{2}}\to n=2\to m=\frac{7}{2}\to n+m=\boxed{\frac{11}{2}}$

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$f^{-۱}(۲)=x \to f(x)=۲ \to \frac{\sqrt{x}+۱}{\sqrt{x}-۱}=۲ \to x=۹$

$f'(۹)=\frac{-۱}{\sqrt{x}(\sqrt{x}-۱)^۲}=\frac{-۱}{۱۲} \to f'^{-۱}_{(x)}$ مشتق $\Rightarrow \boxed{-۱۲}$

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$\frac{f(۰)-f(-۱)}{a-(-۱)}=f(۰)-f(-۱)=-۱۱ \to ۱-(۸(-a+۱))=-۱۱ \to ۸a=-۴ \to a=\frac{-۱}{۲}$

$f'(-۲\times\frac{-۱}{۲})=f'(۱) \to f'(x)=(۳\times(x^۲+۱)^۲\times ۲x)(-\frac{۱}{۲}x+۱)+(x^۲+۱)^۳(-\frac{۱}{۲}) \to f'(۱)=\boxed{-۲۰}$

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$y=\sin x\cos ۲x \to \frac{f(\frac{\pi}{۲})-f(\frac{\pi}{۴})}{\frac{\pi}{۲}-\frac{\pi}{۴}}=\frac{-۱}{\frac{\pi}{۴}}=\frac{-۴}{\pi}$

$y=\sin^۴ x-\cos^۴ x=\sin^۲ x-\cos^۲ x \to \frac{f(\frac{\pi}{۲})-f(\frac{\pi}{۴})}{\frac{\pi}{۲}-\frac{\pi}{۴}}=\frac{۱}{\frac{\pi}{۴}}=\frac{۴}{\pi}$

$\to \frac{\frac{-۴}{\pi}}{\frac{۴}{\pi}}=-۱$ برابر