نام: دینا احدی   شماره کلیف: ۲۳   مقطع: دختران دوازدهم B   ۱۸,۷۵

①

$g'(x) = 2\tan x(1+\tan^2 x) - \sqrt{2}\sin x).f'(\tan^2 x + \sqrt{2}\cos x) \to \tan^2 x + \sqrt{2}\cos x = 2$ ۲

$\to x = \frac{\pi}{4} \to g'(\frac{\pi}{4}) = \sqrt{3} = (2\tan(\frac{\pi}{4})(1+\tan^2(\frac{\pi}{4})) - \sqrt{2}\sin(\frac{\pi}{4}) \times f'(2) \to f'(2) = \boxed{\frac{\sqrt{3}}{3}}$

②

$\cos(\frac{7\pi}{6}) = -\frac{\sqrt{3}}{2}, \sin(\frac{7\pi}{6}) = -\frac{1}{2} \Rightarrow g'(x) = \frac{2\sin x}{(2-\cos^2 x)^2} \Rightarrow g'(\frac{7\pi}{6}) = \frac{76 - 32\sqrt{3}}{169}$

$f'(x) = \frac{(-3\cos^2 x\sin x)(4-\cos^2 x) - (8+\cos^3 x)(-2\sin x\cos x)}{(4-\cos^2 x)^2} \Rightarrow f'(\frac{7\pi}{6}) = \frac{135 - 128\sqrt{3}}{338}$

$\longrightarrow f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = \frac{135 - 128\sqrt{3}}{338} - \frac{304 - 128\sqrt{3}}{338} = \boxed{\frac{-1}{2}}$ ۲

③

$f'_+(x) = 4x\sqrt{ax} + (4x-3)\frac{a}{2\sqrt{ax}}, \quad f'_-(x) = -4x\sqrt{ax} + -(4x-3)\frac{a}{2\sqrt{ax}}$

$f'_+(\frac{3}{4}) - f'_-(\frac{3}{4}) = 2\sqrt{6} \to 8\sqrt{ax} + (8x-6)\frac{a}{2\sqrt{ax}} = 2\sqrt{6} \to 4\sqrt{3a} = 2\sqrt{6} \to a = \boxed{\frac{1}{2}}$ ۲

④

$y = -ax+2 \to f(4) = -4a+2, \ f'(4) = -a \to f(4)+f'(4) = -1 \to -5a = 3 \to a = \frac{3}{5}$

$f'(4) = -a = \boxed{\frac{-3}{5}}$ ۲

⑤

$y = \frac{1}{7}x + \frac{5}{7} \to$ شیب $m = \frac{1}{7} \to y = \frac{ax-1}{3x+1} \to y' = \frac{a+3}{(3x+1)^2} = \frac{1}{7} \to 9x^2+6x-20-7a = 0$ ①

$\frac{ax-1}{3x+1} = \frac{x+5}{7} \to 3x^2+(16-7a)x+12 = 0 \xrightarrow{\times 3} 9x^2+3(16-7a)x+36 = 0$ ② ۱,۵

$\xrightarrow{①,②} x = 1 \to 9+6-20 = 7a \to a = \boxed{\frac{-5}{7}}$

⑥

بر محور x ها مماس $\to f'(2) = 0 \to 3x^2+a = 0 \to a = -12$

$\to f(2) = 0 \to 8+2(-12) = b \to b = -16 \to b-a = \boxed{-4}$ ۲

⑦

$\sin(x^2) \sim x^2 \to f(x) = \sin^n(x^2) \sim (x^2)^n = x^{2n}$

$f(x) = \sin^n(x^2) \to f'(x) = n\sin^{n-1}(x^2).2x.\cos(x^2) \to f'(x) \sim 2nx^{2n-2}.x = 2nx^{2n-1}$

صورت $\Rightarrow f'(x).f(x) \sim 2nx^{4n-1}$ ۱,۷۵

مخرج $\Rightarrow (1-\cos x)^m \sim (\frac{x^2}{2})^m$

$\to \lim_{x\to 0} \frac{f(x)f'(x)}{(1-\cos x)^m} = 2n2^m x^{4n-1-2m}$

حد موجود است → توان صفر شود ور[نه] صفر یا بی نهایت $\quad 4n-1-2m = 0 \to m = 2n-\frac{1}{2}$

$2n2^m = 32\sqrt{2} \to 2n2^{2n-\frac{1}{2}} = 2^{\frac{11}{2}} \to n = 2 \to m = \frac{7}{2} \to n+m = \boxed{\frac{11}{2}}$

۱۱

$f^{-1}(2)=x \to f(x)=2 \to \frac{\sqrt{x}+1}{\sqrt{x}-1}=2 \to x=9$

$f'(9)=\frac{-1}{\sqrt{x}(\sqrt{x}-1)^2}=\frac{-1}{12} \longrightarrow f'^{-1}(x)$ مشتق $\Rightarrow \boxed{-12}$

۲

۹

$\frac{f(0)-f(-1)}{a-(-1)} = f(0)-f(-1)=-11 \to 1-(8(-a+1))=-11 \to 8a=-4 \to a=-\frac{1}{2}$

۱/۵

$f'(-2\times -\frac{1}{2})=f'(1) \to f'(x)=(3\times(x^2+1)^2\times 2x)(-\frac{1}{2}x+1)+(x^2+1)^3(-\frac{1}{2}) \to f'(1)=\boxed{-40}$

۱۰

$y=\sin x\cos 2x \to \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1}{\frac{\pi}{4}}=\frac{-4}{\pi}$

$y=\sin^4 x-\cos^4 x=\sin^2 x-\cos^2 x \to \frac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{1}{\frac{\pi}{4}}=\frac{4}{\pi}$

$\Big\} \to \frac{\frac{-4}{\pi}}{\frac{4}{\pi}}=-1$ برابر

۲

۵

$$\frac{ax-1}{3x+1} = \frac{x}{7}+\frac{5}{7} \rightarrow 7ax-7 = 3x^2+x+15x+5$$

$$\boxed{a=\frac{4}{7}} \qquad \boxed{a=4}$$

$$3x^2+x(16-7a)+12=0 \xrightarrow{\Delta=0} (16-7a)^2-12^2 = (16-7a-12)(16-7a+12)$$

$\boxed{a=4} \rightarrow 3x^2-12x+12 \rightarrow \boxed{x=2}$ ✓

$a=\frac{4}{7} \rightarrow 3x^2+12x+12 \rightarrow x=-2$ ناحیه اول ✗

۷

$$\lim_{x\to 0} \frac{\sin^n(x^2) \times 2x \sin^{n-1}(x^2) \times \cos x^2}{(2\sin^2\frac{x}{2})^m} \overset{\text{هم‌ارزی}}{\rightsquigarrow} \lim_{x\to 0} \frac{(x^2)^n \times n \times (x^2)^{n-1} \times 2x}{(2(\frac{x}{2})^2)^m} = \frac{(x)^{2n+2n-2+1} \times 2n}{(x)^{2m} \times \frac{1}{2}}$$

$$= \frac{2nx^{4n-1}}{\frac{1}{2^m} \times x^{2m}} = 2^{m+1} \times n \times x^{4n-2m-1}$$

$\rightarrow 4n-2m-1=0 \rightarrow n=\frac{2m+1}{4}$

$\rightarrow 2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \rightarrow m=\frac{7}{2},\ n=2 \rightarrow \boxed{2m+n=9}$

۹

$$\frac{f(0)-f(-1)}{0-(-1)} = \frac{1-8(-a+1)}{1} = 8a-7=-11 \rightarrow 8a=-4 \rightarrow \boxed{a=-\frac{1}{2}}$$

$$f'(x) = 3(x^2+1)^2(2x)(ax+1) + a(x^2+1)^3 \xrightarrow{-2a=1} f'(1) = 3(2)^2(2)(\frac{1}{2}) + -\frac{1}{2}(2)^3$$

$$f'(1) = 12-4 = \boxed{8}$$