تینا جعفری پور — تکلیف ۲۳ — دوازدهم دختر B

۱۹/۵ آفرین

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$g(x)=f(\tan^2x+\sqrt{2}\cos x) \Rightarrow g'(x)=f'(\tan^2x+\sqrt{2}\cos x)\times(2\tan x(1+\tan^2x)-\sqrt{2}\sin x)$

$\Rightarrow g'(\frac{\pi}{4})=f'(2)\times(4-1)=\sqrt{3} \Rightarrow \boxed{f'(2)=\frac{\sqrt{3}}{3}}$ ✓

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$f(x)-2g(x)=\frac{(2+\cos x)(4+\cos^2x-2\cos x)}{4-\cos^2x}-\frac{4}{2-\cos x}=\frac{\cos x(\cos x-2)}{2-\cos x}=-\cos x \Rightarrow f'(x)-2g'(x)=\sin x$

$\Rightarrow f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=\sin\frac{7\pi}{6}=\boxed{\frac{-1}{2}}$ ✓

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$x=\frac{3}{4}\begin{cases} f(x)=(4x-3)\sqrt{ax} \Rightarrow f'_+(\frac{3}{4})=4\sqrt{\frac{3}{4}a} & \text{(مشتق حاصل‌ضرب)}\\ f(x)=-(4x-3)\sqrt{ax} \Rightarrow f'_-(\frac{3}{4})=-4\sqrt{\frac{3}{4}a} \end{cases}$

$\Rightarrow \left|4\sqrt{\frac{3}{4}a}-(-4\sqrt{\frac{3}{4}a})\right|=\frac{8\sqrt{3a}}{2}=4\sqrt{3a}=2\sqrt{6}$

$\Rightarrow \boxed{a=\frac{1}{2}}$ ✓

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$y+ax=2 \Rightarrow y=-ax+2 \Rightarrow \begin{cases} f'(4)=-a\\ f(4)=-4a+2\end{cases} \Rightarrow f(4)+f'(4)=-5a+2=-1 \Rightarrow a=\frac{3}{5} \Rightarrow \boxed{f'(4)=-\frac{3}{5}}$ ✓

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$y=\frac{ax-1}{3x+1} \Rightarrow 7y=\frac{7ax-7}{3x+1}=x+5 \Rightarrow 7ax-7=3x^2+16x+5 \Rightarrow 3x^2+(16-7a)x+12=0 \xrightarrow{\Delta=0} (16-7a)^2-4(3)(12)=0$

چون مماس شده ریشه مضاعف دارد

$\Rightarrow 256+49a^2-224a-144=0 \Rightarrow 49a^2-224a+112=0 \xrightarrow{\div 7} 7a^2-32a+16=(7a-4)(a-4)=0 \begin{cases} a=4\\ a=\frac{4}{7}\end{cases}$

$\boxed{a=4} \Rightarrow 3x^2+(16-28)x+12=3x^2-12x+12=0 \xrightarrow{\div 3} x^2-4x+4=(x-2)^2=0 \Rightarrow x=2\ ,\ y=1$ ✓

$a=\frac{4}{7} \Rightarrow 3x^2+(16-4)x+12=3x^2+12x+12=0 \xrightarrow{\div 3} x^2+4x+4=(x+2)^2 \Rightarrow x=-2$ غ ق ق

$y>0$ و $x>0$: ناحیه اول

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$\begin{cases} f(2)=0 \Rightarrow 8+2a-b=0 \Rightarrow b=-16\\ f'(2)=0 \Rightarrow f'(x)=3x^2+a \Rightarrow f'(2)=12+a=0 \Rightarrow a=-12\end{cases} \Rightarrow b-a=-16-(-12)=\boxed{-4}$ ✓

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$f(x)=\sin^n(x^2) \Rightarrow f'(x)=n\sin^{(n-1)}(x^2)\times\cos(x^2)\times 2x$

$\lim_{x\to 0}\frac{f(x)f'(x)}{(1-\cos x)^m}=\lim_{x\to 0}\frac{\sin^n(x^2)\times 2nx\sin^{(n-1)}(x^2)\times\cos(x^2)}{(1-\cos x)^m}=\lim_{x\to 0}\frac{\sin^{(2n-1)}(x^2)\times 2nx\cos(x^2)}{(1-\cos x)^m}\overset{\text{هم‌ارزی}}{=}\lim_{x\to 0}\frac{(x^2)^{2n-1}\times 2nx\times 1}{(1-(1-\frac{x^2}{2}))^m}=\lim_{x\to 0}\frac{x^{4n-1}\times 2n}{(\frac{x^2}{2})^m}=\lim_{x\to 0}2^m\times 2n=32\sqrt{2}$

$\begin{cases} 4n-1=2m \Rightarrow m=2n-\frac{1}{2}=\frac{7}{2}\\ 2^{m+1}\times n=32\sqrt{2} \Rightarrow 2^{2n+\frac{1}{2}}\times n=2^{2n}\times\sqrt{2}\times n=32\sqrt{2} \Rightarrow 2^{2n}\times n=32 \Rightarrow n=2\end{cases}$

$\Rightarrow 2m+n=2(\frac{7}{2})+2=\boxed{9}$ ✓

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$f(x)=\frac{\sqrt{x}+1}{\sqrt{x}-1} \Rightarrow f'(x)=\frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1)-\frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2}=\frac{1}{2\sqrt{x}}\frac{(\sqrt{x}-1-\sqrt{x}-1)}{(\sqrt{x}-1)^2}=\frac{-1}{\sqrt{x}(\sqrt{x}-1)^2} \Rightarrow f'(2)=\frac{-1}{\sqrt{2}(\sqrt{2}-1)^2}$

$\Rightarrow (f^{-1}(2))'=-\frac{\sqrt{2}(3-2\sqrt{2})}{1}=\boxed{4-3\sqrt{2}}$

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$f(x)=(x^2+1)^3(ax+1)$, $\frac{f(0)-f(-1)}{0-(-1)}=\frac{1-8(-a+1)}{1}=8a-7=-11 \Rightarrow a=-\frac{1}{2} \Rightarrow f(x)=(x^2+1)^3(-\frac{1}{2}x+1)$ ✓

$\Rightarrow f'(x)=3(x^2+1)^2\times 2x(-\frac{1}{2}x+1)+(-\frac{1}{2})(x^2+1)^3 \Rightarrow f'(1)=3\times 4\times 2\times\frac{1}{2}+(-\frac{1}{2})\times 8=\boxed{8}$

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$y=\sin x.\cos 2x$

$y=\sin^4x-\cos^4x=-\cos 2x$

$\frac{\sin\frac{\pi}{2}\cos\pi-\sin\frac{\pi}{4}\cos\frac{\pi}{2}}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{-1-0}{\frac{\pi}{4}}=-\frac{4}{\pi}$

$\frac{-\cos\pi-(-\cos\frac{\pi}{2})}{\frac{\pi}{2}-\frac{\pi}{4}}=\frac{1+0}{\frac{\pi}{4}}=\frac{4}{\pi}$

$\Rightarrow \frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=\boxed{-1}$ ✓

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$f^{-1}(۲) = t \rightarrow f(t) = ۲ \longrightarrow \sqrt{t} + ۱ = ۲\sqrt{t} - ۲ \longrightarrow \sqrt{t} = ۳ \longrightarrow t = ۹$

$f^{-1\,\prime}(۲) = \dfrac{۱}{f'(۹)}$, $f'(x) = \dfrac{-۲}{(\sqrt{x} - ۱)^۲} \times \dfrac{۱}{۲\sqrt{x}}$

$f'(۹) = \dfrac{-۲}{(۲)^۲} \times \dfrac{۱}{۶} = -\dfrac{۱}{۱۲} \longrightarrow \boxed{f^{-1\,\prime}(۲) = -۱۲}$