Date No

اولامای کیا - تکلیف ۲۳

۱. $g'(x) = (2(1+\tan^2 x) - \sqrt{3}\sin x)\, f'(\tan^2 x + \sqrt{3}\cos x)$

$x = \frac{\pi}{4} \to \underbrace{g'(\frac{\pi}{4})}_{\sqrt{3}} = \underbrace{(2(\frac{3}{2}) - \sqrt{2}\times\frac{\sqrt{2}}{2})}_{2}\, f'\underbrace{(1+1)}_{2} \Rightarrow f'(2) = \frac{\sqrt{3}}{2}$

۲. $f(x) = \frac{\cancel{(\cos x+2)}(\cos^2 x - 2\cos x + 4)}{\cancel{(\cos x+2)}(2-\cos x)} \Rightarrow f'(x) \overset{hop}{=} \frac{-2\cos x\sin x + 2\sin x}{\sin x}$

$= -2\cos x + 2$ $\quad g'(x) \overset{hop}{=} \frac{1}{2+\sin x}$ $\quad f'(\frac{7\pi}{6}) = -2(-\frac{\sqrt{3}}{2}) + 2 = 2+\sqrt{3}$

$g'(\frac{7\pi}{6}) = \frac{2}{3} \Rightarrow f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = 2+\sqrt{3} - \frac{4}{3} = \frac{2}{3} + \sqrt{3}$

۳. $x \to \frac{3}{4}^+ \to f(x) = (4x-3)\sqrt{ax} \Rightarrow f'(x) = 4\sqrt{ax} + \frac{a(4x-3)}{2\sqrt{ax}}$

$x \to \frac{3}{4}^- \to f(x) = (3-4x)\sqrt{ax} \Rightarrow f'(x) = -4\sqrt{ax} + \frac{a(3-4x)}{2\sqrt{ax}}$

اختلاف $\to 8\sqrt{ax} + \frac{a(4x-3)}{\sqrt{ax}} = 2\sqrt{6} \xrightarrow{x=\frac{3}{4}} 8\sqrt{\frac{3}{4}a} = 2\sqrt{6} \Rightarrow a = 2$

۴. معادله
شیب برابر: $y = -ax+2 \to m = -a \quad f'(4) = -a$
مقدار برابر: $x = 4 \to y = 2-4a \quad f(4) = 2-4a$

$\Rightarrow 2-4a-a = 2-5a = -1 \Rightarrow 5a = 3 \Rightarrow a = \frac{3}{5} \quad f'(4) = -\frac{3}{5}$

۵. $y = \frac{x+5}{7} \Rightarrow m = \frac{1}{7} \quad f'(x) = \frac{a+3}{(3x+1)^2} \Rightarrow \frac{a+3}{(3x+1)^2} = \frac{1}{7}$

مقدارها باید برابر باشه $\to \frac{x+5}{7} = \frac{ax-1}{3x+1} \Rightarrow 3x^2 + 16x + x + 5 = 7ax - 7$

$3x^2 + (16-7a)x + 12 = 0$

چون معادله سهمی ریشه مضاعف داره

$\Delta = 0 \to (16-7a)^2 - 4(3)(12) = 0$

$256 + 49a^2 - 224a - 144 = 49a^2 - 224a + 112 = 0$

$7a^2 - 32a + 16 = 0 \to a = \frac{32 \pm 24}{14} = 4,\ \frac{8}{14}$

if $a = 4$ نقطه تماس: $(2, 1)$ ✓ $\quad$ if $a = \frac{8}{14}$: نقطه $(\frac{4}{3}, \frac{19}{21})$ ✓

dotnote

۶. $f'(x) = 3x^2 + a \xrightarrow{x=2} 12 + a$ مماس بر محور x $\Rightarrow m = 0 \rightarrow 12 + a = 0$

$a = -12$

$x = 2 \rightarrow f(x) = 0 \Rightarrow (2)^3 - 12(2) - b = 0 \Rightarrow b = -16$

$b - a = -16 - (-12) = -4$

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۷. $f'(x) = \underbrace{n\sin^{n-1}(x^2)}\,\underbrace{\cos(x^2)}\,(2x)$ $\star\ \sin u \sim u$ $\star\ 1-\cos u \sim \frac{u^2}{2}$

$\xrightarrow[x\to 0]{\text{هم ارزی}} n(x^2)^{n-1} \times 1 \times 2x = 2n x^{2n-1}$

$\lim_{x\to 0} \frac{f(x)\,f'(x)}{(1-\cos x)^m} \xrightarrow{\text{هم ارزی}} \frac{x^{2n} \times 2n x^{2n-1}}{(\frac{x^2}{2})^m} = n \times 2^{m+1}\, x^{4n-2m-1}$

به عدد ثابت، توان x صفر شد

$4n - 2m = 1$ $2^{m+1} \times n = 32\sqrt{2} = 2^{\frac{11}{2}}$ $n = 1$, $m+1 = \frac{11}{2} \Rightarrow m = \frac{9}{2}$

$2m + n = 2(\frac{9}{2}) + 1 = 10$

ببخشید این سوال راه حل سریع‌تری نداره؟

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۸. $f^{-1}(2) = ? \rightarrow f(?) = 2 \rightarrow \sqrt{x} + 1 = 2\sqrt{x} - 2 \Rightarrow \sqrt{x} = 3 \Rightarrow x = 9 \Rightarrow$

$f(9) = 2$

$f'(x) = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - \frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2} \Rightarrow f'(9) = \frac{\frac{2}{6} - \frac{4}{6}}{4} = \frac{-1}{12}$

مشتق تابع معکوس = معکوس مشتق تابع $\Leftarrow$ شیب $f^{-1}(2) = -12$

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۹. $f(-1) = 8(-a+1)$ $f(0) = 1 \rightarrow$ آهنگ تغییر متوسط $= \frac{f(0) - f(-1)}{0-(-1)} = \frac{1 + 8a - 8}{1}$

$8a - 7 = -11 \Rightarrow a = \frac{-1}{2} \rightarrow -2a = 1$

آهنگ تغییر لحظه‌ای $\Leftarrow f'(x) = 3(x^2+1)^2(2x)(ax+1) + a(x^2+1)^3$

$3(4)(2)(\frac{1}{2}) + (-\frac{1}{2})(8) = 12 - 4 = 8$

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۱۰. $f(\frac{\pi}{2}) = -1$ $f(\frac{\pi}{4}) = 0 \Rightarrow$ آهنگ متوسط: $\frac{-1-0}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{-4}{\pi}$

$g(\frac{\pi}{2}) = 1$ $g(\frac{\pi}{4}) = 0 \Rightarrow$ آهنگ متوسط: $\frac{1-0}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{4}{\pi}$

(۱-) برابر