نام و نام خانوادگی: ... آدرینا جمشیدی	شماره کلاس: ۱۲	کلاس: 12B

نمره: ۱۹٫۵

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$g(x)=f(\tan^2 x+\sqrt{2}\cos x)$

$\Rightarrow g'(x)=\big(2(1+\tan^2 x)\tan x-\sqrt{2}\sin x\big)f'(\tan^2 x+\sqrt{2}\cos x)$

$g'(\frac{\pi}{4})=\big(2(1+1)(1)-1\big)f'\big(1+\sqrt{2}(\frac{\sqrt{2}}{2})\big) \Rightarrow \sqrt{3}=3f'(2)$

$\Rightarrow \boxed{f'(2)=\frac{\sqrt{3}}{3}}$

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$f(x)=\frac{8+\cos^3 x}{4-\cos^2 x}=\frac{(2+\cos x)(4-2\cos x+\cos^2 x)}{\cancel{(2+\cos x)}(2-\cos x)}=\frac{\cos^2 x-2\cos x+4}{2-\cos x}$

$g(x)=\frac{2}{2-\cos x}$

$f'(\frac{7\pi}{6})-2g'(\frac{7\pi}{6})=\big(f(\frac{7\pi}{6})-2g(\frac{7\pi}{6})\big)'=\Big(\frac{\cos^2 x-2\cos x+4}{2-\cos x}-\frac{4}{2-\cos x}\Big)'$

$=(-\cos x)'=\sin x=\sin\frac{7\pi}{6}=-\frac{1}{2}$

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$f(x)=|4x-3|\sqrt{ax}$

$f'_+(\frac{3}{4})=\big((4x-3)\sqrt{ax}\big)'=4\sqrt{\frac{3}{4}a}$

$f'_-(\frac{3}{4})=\big((3-4x)\sqrt{ax}\big)'=-4\sqrt{\frac{3}{4}a}$

$|f'_+(\frac{3}{4})-f'_-(\frac{3}{4})|=8\sqrt{ax}=8\sqrt{\frac{3}{4}a}=4\sqrt{3a}=2\sqrt{12a}\Rightarrow a=\frac{1}{2}$

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$y+ax=2 \Rightarrow y=-ax+2$

$f(4)+f'(4)=-1$ $\quad f'(4)=-\frac{3}{5}$

$-4a+2-a=-1$

$-5a=-3 \quad a=\frac{3}{5}$

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$7y-x=5 \qquad y=\frac{ax-1}{3x+1}$

$y=\frac{x}{7}+\frac{5}{7}$

$y'=\frac{1}{7}$

$M(\alpha,\frac{\alpha+5}{7})$

$\frac{ax-1}{3x+1}=\frac{x+5}{7} \Rightarrow 7ax-7=3x^2+x+15x+5$

$\Rightarrow 3x^2+(16-7a)x+12=0$

$\Delta=0 \Rightarrow 16-7a=12 \Rightarrow a=\frac{4}{7}$ غ ق ق

$16-7a=-12 \Rightarrow \boxed{a=4}$

[illegible]

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$f(x)=x^3+ax-b$

$f(2)=8+2a-b=0 \quad \boxed{b=-16} \qquad b-a=-16+12=-4$ ۶

$f'(x)=3x^2+a \Rightarrow f'(2)=12+a=0 \quad \boxed{a=-12}$ ۲

$f(x)=\sin^n(x^2) \qquad 2m+n=9$

$\lim\limits_{x\to 0}\dfrac{f(x)f'(x)}{(1-\cos x)^m}=32\sqrt{2}$

$\lim\limits_{x\to 0}\dfrac{(\sin^n(x^2))(2x(n)(\cos x^2)(\sin^{n-1}x^2))}{(1-\cos x)^m}=\lim\limits_{x\to 0}\dfrac{(x^{2n})(2x)(n)\left(1-\frac{x^4}{2}\right)x^{2n-2}}{\left(\frac{x^2}{2}\right)^m}$ ۱٫۵ ۷

$=\dfrac{(x^{4n-1})(2n)2^m}{x^{2m}} \qquad 4n-1=2m \qquad 2^{m+1}n=32\sqrt{2}$

$f(x)=\dfrac{\sqrt{x}+1}{\sqrt{x}-1}=2 \qquad f'(x)=\dfrac{1}{2\sqrt{x}}\left(\dfrac{-2}{(\sqrt{x}-1)^2}\right)$

$\sqrt{x}+1=2\sqrt{x}-2$
$3=\sqrt{x}\Rightarrow x=9 \quad \Rightarrow f'(9)=\dfrac{1}{2(3)}\left(\dfrac{-2}{(2)^2}\right)=-\dfrac{1}{12}$ ۸

$f^{-1\,\prime}(2)=\dfrac{1}{f'(9)}=-12$ ۲

$f(x)=(x^2+1)^3(ax+1) \quad [-1,0] \qquad a=-15=1$

$f(0)=(0+1)^3(a(0)+1)=1 \qquad \dfrac{f(b)-f(a)}{b-a}=\dfrac{1+8a-8}{0+1}=8a-7=-11$ ۹

$a=-\frac{1}{2}$

$f(-1)=(1+1)^3(-a+1)=-8a+8$

$f'(x)=3(2x)(x^2+1)^2\left(-\frac{x}{2}+1\right)+\left(-\frac{1}{2}\right)(x^2+1)^3$ ۲

$=3(2)(2)^2\left(\cancel{\tfrac{1}{2}}\right)+\left(-\frac{1}{2}\right)(2)^3=12-4=8$

$g=\sin x\cos 2x \qquad \left[\frac{\pi}{4},\frac{\pi}{2}\right] \qquad g=\sin^4x-\cos^4x$

$g=-\cos 2x$

$\dfrac{f(b)-f(a)}{b-a}=\dfrac{-1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=-\dfrac{4}{\pi} \qquad \dfrac{1-0}{\frac{\pi}{2}-\frac{\pi}{4}}=\dfrac{4}{\pi}$ ۱۰

$\dfrac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$ ۲

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$$\lim_{x\to ۰} \frac{\sin^{n}(x^{۲}) \times ۲x\sin^{n-۱}(x^{۲}) \times \cos x^{۲}}{(۲\sin^{۲}\frac{x}{۲})^{m}} \xrightarrow{\text{هم ارزی}} \lim_{x\to ۰} \frac{(x^{۲})^{n} \times n \times (x^{۲})^{n-۱} \times ۲x}{(۲(\frac{x}{۲})^{۲})^{m}} = \frac{(x)^{۲n+۲n-۲+۱} \times ۲n}{(x)^{۲m} \times \frac{۱}{۲}}$$

$$= \frac{۲nx^{۴n-۱}}{\frac{۱}{۲^{m}} \times x^{۲m}} = ۲^{m+۱} \times n \times x^{۴n-۲m-۱}$$

$$\to ۴n-۲m-۱=۰ \longrightarrow n = \frac{۲m+۱}{۴}$$

$$\to ۲^{m+۱} \times \frac{۲m+۱}{۴} = ۳۲\sqrt{۲} \longrightarrow m = \frac{۷}{۲},\ n = ۲ \longrightarrow \boxed{۲m+n=۹}$$