[illegible]

[illegible]

(18)

$f^{-1}(x)\big|_{2} = f(x)\big|_{z} \Rightarrow \frac{\sqrt{x}+1}{\sqrt{x}-1} = 2 \Rightarrow 2\sqrt{x}-2 = \sqrt{x}+1$

$\sqrt{x} = 3 \Rightarrow x = 9$

$z = 9$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - \frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}+1)^2} \qquad f'(9) = \frac{\frac{1}{6}\times 2 - \frac{1}{6}\times 4}{16}$$

$$f'(9) = \frac{-\frac{1}{3}}{\frac{16}{1}} \Rightarrow f'(9) = -\frac{1}{48} \qquad \left(f^{-1}(2)\right)' = -48 \,]$$

(19)

$$\frac{f(0) - f(-1)}{0-(-1)} = -11 \qquad \frac{1-(8)(-a+1)}{1} = -11$$

$$\Rightarrow 1 + 8a - 8 = -11 \Rightarrow 8a = -4 \Rightarrow a = -\frac{1}{2} \,]$$

$$f'(x) = 3(x^2+1)^2(2x)(ax+1) + (x^2+1)^3(a)$$

$$x = 1,\ a = -\tfrac{1}{2}\} \Rightarrow f'(1) = 24\left(-\tfrac{1}{2}\times 1 + 1\right) + 8\left(-\tfrac{1}{2}\right) = 12 - 4 = 8 \,]$$

(20)

$f(x) = \sin x \cos 2x \qquad \frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{-1-0}{\frac{\pi}{4}} = -\frac{4}{\pi}$

$g(x) = \sin^4 x - \cos^4 x \qquad \frac{g(\frac{\pi}{2}) - g(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{1-0}{\frac{\pi}{4}} = \frac{4}{\pi}$

$$\frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$$ برابر

حل همه روزه‌های یک ترکیب آنی :

سوگند فرزادیان

(1)

$g'(x) = \left(2\tan x\,(1+\tan^2 x) - \sqrt{2}\sin x\right)\left(f'(\tan^2 x + \sqrt{2}\cos x)\right)$

$g'\left(\frac{\pi}{4}\right) = 2\times 1\,(1+1) - \sqrt{2}\times\frac{\sqrt{2}}{2}\left(f'\left(1+\sqrt{2}\times\frac{\sqrt{2}}{2}\right)\right) = (4-1)\left(f'(2)\right) \Rightarrow 3f'(2) = \sqrt{3}$

$f'(2) = \frac{\sqrt{3}}{3}$

(2)

$f(x) = \frac{(2+\cos x)(4+\cos^2 x - 2\cos x)}{(2-\cos x)(2+\cos x)} = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x}$

$f - g = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x} - \frac{2\times 2}{2-\cos x} = \frac{\cos x(\cos x - 2)}{2-\cos x} = -\cos x$

$(f-g)' = -(-\sin x) = \sin x \Rightarrow \sin\frac{7\pi}{6} = -\frac{1}{2}$

(3)

$f'(x) = \frac{f(x) - f\left(\frac{3}{4}\right)}{x-\frac{3}{4}} \Rightarrow f'\left(\frac{3}{4}\right) = \frac{|4x-3|\sqrt{ax} - 0}{x-\frac{3}{4}}$

$f'_+\left(\frac{3}{4}\right) = \frac{4x-3\sqrt{ax}}{x-\frac{3}{4}}, \quad f'_-\left(\frac{3}{4}\right) = \frac{-(4x-3)\sqrt{ax}}{x-\frac{3}{4}} \Rightarrow f'_+\left(\frac{3}{4}\right) - f'_-\left(\frac{3}{4}\right) =$

$\frac{2(4x-3)\sqrt{ax}}{x-\frac{3}{4}} = \frac{2\times 4\left(x-\frac{3}{4}\right)\sqrt{ax}}{x-\frac{3}{4}} = 2\sqrt{6} \Rightarrow 8\sqrt{a\times\frac{3}{4}} = 2\sqrt{6} \Rightarrow a = 2$

(4)

$y = -ax+2 \Rightarrow f(4) = -4a+2 \quad f'(4) = -a \Rightarrow f'(4) + f(4) =$

$-4a+2-a = -1 \Rightarrow -5a = -3 \Rightarrow a = \frac{3}{5} \Rightarrow f'(4) = -\frac{3}{5}$

(5) معادله را از دو طرف باید ریشه مضاعف داشته باشد.

$\frac{ax-1}{3x+1} = \frac{5+x}{7} \Rightarrow 3x^2+16x+5 = 7ax - 7$

$\Rightarrow 3x^2 - (16-7a)x + 12 = 0 \Rightarrow \Delta = 0 \Rightarrow (16-7a)^2 - 4(3)(12) = 0$

$(16-7a)^2 = (12)^2 \Rightarrow 16-7a = 12 \Rightarrow a = \frac{4}{7}$

$16-7a = -12 \Rightarrow a = 4 \quad 3x^2-12x+12 = 0 \quad 3(x-2)^2 = 0$

$x = 2$ نامعیراول است $x=2$ پس $a=4$ قق است

(6) $x=2$ هم فرمش و هم مشتق تابع را صفر می‌کند

$f(2) = 8+2a-b = 0$

$f'(x) = 3x^2 + a \Rightarrow f'(2) = 3\times 4 + a = 0 \Rightarrow a = -12$

$8-24-b = 0 \Rightarrow b = -16$

$b - a = -16 - (-12) = -4$

(7)

$\lim_{x\to 0} \frac{\cos^2 x\; 2nx\sin^{n-1}(x^2)\times\sin^n(x^2)}{(1-\cos x)^m} \overset{\text{هم‌ارزی}}{=} \lim_{x\to 0} \frac{2nx\,(x^2)^{n-1}\times(x^2)^n\times\left(1-\frac{x^4}{2}\right)}{\frac{x^{2m}}{4}}$