نمره [illegible] :

سوگند زارعی

(۱۸)

$f'(x)\big|_2 \quad f(x)\big|_2^{?} \Rightarrow \frac{\sqrt{x}+1}{\sqrt{x}-1} = 2 \Rightarrow 2\sqrt{x}-2 = \sqrt{x}+1$

$\sqrt{x} = 3 \Rightarrow x = 9$

$= 9$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1) - \frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}+1)^2} \qquad f'(9) = \frac{\frac{1}{6}\times 2 - \frac{1}{6}\times 4}{16}$$

۱/۵

$$f'(9) = \frac{-\frac{1}{3}}{\frac{16}{1}} \Rightarrow f'(9) = -\frac{1}{48} \qquad \left(f^{-1}(2)\right)' = -48$$

(۱۹)

$$\frac{f(0) - f(-1)}{0-(-1)} = -11 \qquad \frac{1 - (8)(-a+1)}{1} = -11$$

$\Rightarrow 1 + 8a - 8 = -11 \Rightarrow 8a = -4 \Rightarrow a = -\frac{1}{2}$

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$$f'(x) = 3(x^2+1)^2(2x)(ax+1) + (x^2+1)^3(a)$$

$x = 1 \Rightarrow f'(1) = 24\left(-\frac{1}{2}\times 1+1\right) + 8\left(-\frac{1}{2}\right) = 12 - 4 = 8$

(۱۵)

$f(x) = \sin x \cos 2x \qquad \frac{f(\frac{\pi}{2}) - f(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{-1-0}{\frac{\pi}{4}} = -\frac{4}{\pi}$

$g(x) = \sin^4 x - \cos^4 x \qquad \frac{g(\frac{\pi}{2}) - g(\frac{\pi}{4})}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{1-0}{\frac{\pi}{4}} = \frac{4}{\pi}$

$\frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$ برابر

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ره نقص حل مسیر ... های ترکیب آلی :

سوگند ارغال

18,5

(1) $g'(x) = (2\tan x(1+\tan^2 x) - \sqrt{3}\sin x)(f'(\tan^2 x + \sqrt{3}\cos x))$

$g'(\frac{\pi}{4}) = 2\times1(1+1) - \sqrt{2}\times\frac{\sqrt{2}}{2}(f'(1+\sqrt{2}\times\frac{\sqrt{2}}{2})) = (4-1)(f'(2)) \Rightarrow 3f'(2) = \sqrt{3}$

$f'(2) = \frac{\sqrt{3}}{3}$ ✓ 2

(2) $f(x) = \frac{(2+\cos x)(4+\cos^2 x - 2\cos x)}{(2-\cos x)(2+\cos x)} = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x}$

$f - g = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x} - \frac{2\times 2}{2-\cos x} = \frac{\cos x(\cos x - 2)}{2-\cos x} = -\cos x$ 2

$(f-g)' = -(-\sin x) = \sin x \Rightarrow \sin\frac{7\pi}{6} = -\frac{1}{2}$] ✓

(3) $f'(x) = \frac{f(x) - f(\frac{3}{4})}{x-\frac{3}{4}} \Rightarrow f'(\frac{3}{4}) = \frac{|4x-3|\sqrt{ax} - 0}{x-\frac{3}{4}}$

$f'_+(\frac{3}{4}) = \frac{4x-3\sqrt{ax}}{x-\frac{3}{4}}$, $f'_-(\frac{3}{4}) = \frac{-(4x-3)\sqrt{ax}}{x-\frac{3}{4}} \Rightarrow f'_+(\frac{3}{4}) - f'_-(\frac{3}{4}) =$ 1,75

$\frac{2(4x-3)\sqrt{ax}}{x-\frac{3}{4}} = \frac{2\times4(x-\frac{3}{4})\sqrt{ax}}{x-\frac{3}{4}} = 2\sqrt{6} \Rightarrow 8\sqrt{a\frac{3}{4}} = 2\sqrt{6} \Rightarrow a = 2$]

(4) $y = -ax+2 \Rightarrow f(4) = -4a+2$ $f'(4) = -a \Rightarrow f'(4) + f(4) =$

$-4a+2-a = -1 \Rightarrow -5a = -3 \Rightarrow a = \frac{3}{5} \Rightarrow f'(4) = -\frac{3}{5}$] 2

(5) معادله ی (خودر) ۲ منحنی باید ریشه مضاعف داشته باشد.

$\frac{ax-1}{3x+1} = \frac{5+x}{7} \Rightarrow 3x^2+16x+5 = 7ax - 7$

$\Rightarrow 3x^2 - (16-7a)x + 12 = 0 \Rightarrow \Delta = 0 \Rightarrow (16-7a)^2 - 4(3)(12) = 0$

$(16-7a)^2 = (12)^2 \Rightarrow 16 - 7a = 12 \Rightarrow a = \frac{4}{7}$

$16 - 7a = -12 \Rightarrow a = 4$ — $3x^2 - 12x + 12 = 0$ $3(x-2)^2 = 0$ 2

$x = 2$ ناحیه اول است

پس $a = 4$ ق ق است

(6) $x = 2$ هم فرمش و هم مشتق تابع را صفر می کند

$f(2) = 8 + 2a - b = 0$

$f'(x) = 3x^2 + a \Rightarrow f'(2) = 3\times4 + a = 0 \Rightarrow a = -12$

$8 - 24 - b = 0 \Rightarrow b = -16$

$b - a = -16 - (-12) = -4$] 2

(7) $\lim_{x\to 0} \frac{\cos^2 x \cdot 2nx\sin^{n-1}(x^2) \times \sin^n(x^2)}{(1-\cos x)^m}$ $\underset{\text{هم ارزی}}{=}$ $\lim_{x\to 0} \frac{2xn\,(x^2)^{n-1}\times(x^2)^n\times(1-\frac{x^4}{2})}{\frac{x^{2m}}{4}}$

1,25

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$x \to \frac{3}{4}^{+} \rightsquigarrow f(x) = 4x - 3\sqrt{\frac{3a}{4}} \xrightarrow[\text{صفرشونده}]{\text{مشتق عبر}} f'(x) = \frac{4\sqrt{3a}}{2} = 2\sqrt{3a}$

$x \to \frac{3}{4}^{-} \rightsquigarrow f(x) = -4x + 3\sqrt{\frac{3a}{4}} \xrightarrow[\text{صفرشونده}]{\text{مشتق عبر}} f'(x) = \frac{-4\sqrt{3a}}{2} = -2\sqrt{3a}$

$4\sqrt{3a} = 2\sqrt{6} \to \sqrt{3a} = \frac{\sqrt{6}}{2} \to \boxed{a = \frac{1}{2}}$

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$$\lim_{x \to 0} \frac{\sin^n(x^2) \times 2x \sin^{n-1}(x^2) \times \cos x^2}{(2\sin^2\frac{x}{2})^m} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0} \frac{(x^2)^n \times n \times (x^2)^{n-1} \times 2x}{(2(\frac{x}{2})^2)^m} = \frac{(x)^{2n+2n-2+1} \times 2n}{(x)^{2m} \times \frac{1}{2}}$$

$$= \frac{2nx^{4n-1}}{\frac{1}{2^m} \times x^{2m}} = 2^{m+1} \times n \times x^{4n-2m-1}$$

$\rightsquigarrow 4n - 2m - 1 = 0 \to n = \frac{2m+1}{4}$

در $2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \to m = \frac{7}{2}, \ n = 2 \to \boxed{2m + n = 9}$

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$f^{-1}(2) = t \to f(t) = 2 \longrightarrow \sqrt{t} + 1 = 2\sqrt{t} - 2 \to \sqrt{t} = 3 \to t = 9$

${f^{-1}}'(2) = \frac{1}{f'(9)}$ و $f'(x) = \frac{-2}{(\sqrt{x} - 1)^2} \times \frac{1}{2\sqrt{x}}$

$f'(9) = \frac{-2}{(2)^2} \times \frac{1}{6} = -\frac{1}{12} \longrightarrow \boxed{{f^{-1}}'(2) = -12}$