راه حل ششم

$$g'(x) = f'(\tan^2 x + \sqrt{2}\cos x) \times (2\tan x(1+\tan^2 x) - \sqrt{2}\sin x) \quad (1)$$

$$\underbrace{g\left(\frac{\pi}{4}\right)}_{\sqrt{2}} = \underbrace{f'(1+1)}_{f'(2)} \times \underbrace{(2(2)-1)}_{2} \Rightarrow f'(2) = \frac{\sqrt{2}}{2}$$

(٢)

$$\left(f\left(\frac{\pi}{4}\right) - 2g\left(\frac{\pi}{4}\right)\right)' = \left(\frac{\cos^2 x - 2\cos x + 1}{2 - \cos x} - \frac{1}{2 - \cos x}\right)'$$

$$= (-\cos x)' = \sin x \downarrow_{\frac{\pi}{4}} = \boxed{-\frac{1}{2}}$$

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(٤)

$$f'(x) = 2\sqrt{ax} + (2x - 1)\frac{a}{2\sqrt{ax}} \Rightarrow f'\left(\frac{1}{2}\right) = 2\sqrt{2a}$$

$\frac{1}{2}^+$

$\frac{1}{2}^-$

$$f'(x) = -2\sqrt{ax} + (1 - 2x)\frac{a}{2\sqrt{ax}} \Rightarrow f'\left(\frac{1}{2}\right) = -2\sqrt{2a}$$

$$2\sqrt{2a} = 2\sqrt{2} \Rightarrow \sqrt{2a} = \frac{\sqrt{2}}{2} \Rightarrow \boxed{a = \frac{1}{2}}$$

(۴)

$f'(۴)=a$

$f(۴)=-۴a+۲$

$-۳a+۲=-۱ \Rightarrow \boxed{a=۱}$

$\boxed{f'(۴)=۱}$

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(۵)

$\left| \begin{matrix} t \\ ۲t+۵ \end{matrix} \right. \longrightarrow$ تماس

$\dfrac{ax-۱}{۳x+۱}=\dfrac{۱}{۷}x+\dfrac{۵}{۷} \Rightarrow ۳t^۲-(۱۶-۷a)t+۱۲=۰$

$y'=\dfrac{a+۳}{(۳x+۱)^۲} \xRightarrow[y'=۰]{x=t} ۷a+۲۱=(۳t+۱)^۲ \Rightarrow$

$-t^۳-t^۲-۲t+۴=۰$

$\Rightarrow -(t-۱)(t+۲)^۲=۰ \rightarrow t=۱,\ t=-۲$

$۷a+۲۱=۱۶ \Rightarrow \boxed{a=\dfrac{-۵}{۷}}$

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(۶)

$y=۰$

$f'(x)=۳x^۲+a \xrightarrow[f'(x)=۰]{x=۲} ۱۲+a=۰ \Rightarrow a=-۱۲$

$f(۲)=۰ \Rightarrow ۸+۲a-b=۰ \Rightarrow b=-۱۶$

$\boxed{b-a=-۴}$

(1)

$$f^{-1}(x) = \left(\frac{x+1}{x-1}\right)^2 \Longrightarrow (f^{-1})'(x) = 2\left(\frac{x+1}{x-1}\right)\left(\frac{-2}{(x-1)^2}\right)$$

$$\Longrightarrow (f^{-1})'(2) = -12$$

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(2)

$$\frac{f(0) - f(-1)}{1} = -11 \Rightarrow 1 - (-10a+1) = -11$$

$$\Longrightarrow a = \frac{1}{2}$$

$$f'(x) = 4(x^2+1)^3(2x)(ax+1) + (x^2+1)^4 a$$

$$\Rightarrow f'(-1) = -1$$

(10)

$$\frac{\sin\frac{\pi}{6}\cos\pi - \sin\frac{\pi}{2}\cos\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{-2}{\pi}$$

(1- برابر)

$$\frac{\sin^2\frac{\pi}{6} - \cos^2\frac{\pi}{6} - \sin\frac{3\pi}{2} + \cos^2\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{2}{\pi}$$