۱۴٫۵

سانا ستاری

$$g'(x) = f'(\tan^2 x + \sqrt{2}\cos x) \times (2\tan x(1+\tan^2 x) - \sqrt{2}\sin x) \quad 1$$

$$\underbrace{g'\left(\frac{\pi}{4}\right)}_{\sqrt{2}} = \underbrace{f'(1+1)}_{f'(2)} \times \left(\underbrace{2(2) - 1}_{3}\right) \Longrightarrow f'(2) = \frac{\sqrt{2}}{3}$$

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$$\left(f\left(\frac{۷\pi}{۶}\right) - ۲g\left(\frac{۷\pi}{۶}\right)\right)' = \left(\frac{\cos^۲ x - ۴\cos x + ۴}{۲-\cos x} - \frac{۴}{۲-\cos x}\right)'$$

$$= (-\cos x)' = \sin \underset{\downarrow\, \frac{۷\pi}{۶}}{x} = -\frac{۱}{۲}$$

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$\frac{۳}{۲}^{+}$: $$f'(x) = ۴\sqrt{ax} + (۴x-۶)\frac{a}{۲\sqrt{ax}} \Rightarrow f'\left(\frac{۳}{۲}\right) = ۲\sqrt{۳a}$$

$\frac{۳}{۲}^{-}$: $$f'(x) = -۴\sqrt{ax} + (۶-۴x)\frac{a}{۲\sqrt{ax}} \Rightarrow f'\left(\frac{۳}{۲}\right) = -۲\sqrt{۳a}$$

$$۴\sqrt{۳a} = ۲\sqrt{۶} \Rightarrow \sqrt{۳a} = \frac{\sqrt{۶}}{۲} \Rightarrow a = \frac{۱}{۲}$$

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$f'(۴)=a$

$f(۴)=-۴a+۲$ $\quad -۳a+۲=-۱ \Rightarrow a=۱$

$\boxed{f'(۴)=۱}$

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$\left|\begin{matrix} t \\ \frac{۱}{۷}t+\frac{۵}{۷} \end{matrix}\right. \longrightarrow$ نقطه تماس

$\frac{ax-۱}{۳x+۱}=\frac{۱}{۷}x+\frac{۵}{۷} \Rightarrow ۳t^۲-(۱۶-۷a)t+۱۲=۰$

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$y'=\frac{a+۳}{(۳x+۱)^۲} \xRightarrow[y'=۰]{x=t} ۷a+۲۱=(۳t+۱)^۲ \Rightarrow$

$-t^۳-t^۲-۲t+۴=۰$

$\Rightarrow -(t-۱)(t+۲)^۲=۰ \to t=۱,\ t=-۲$ غ ق ق

$۷a+۲۱=۱۶ \Rightarrow a=\frac{-۵}{۷}$

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$y=۰$ $\quad f'(x)=۳x^۲+a \xrightarrow[f'(x)=۰]{x=۲} ۱۲+a=۰ \Rightarrow a=-۱۲$

$f(۲)=۰ \Rightarrow ۸+۲a-b=۰ \Rightarrow b=-۱۶$

$b-a=-۴$

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$$f^{-1}(x) = \left(\frac{1+x}{x-1}\right)^{2} \Rightarrow (f^{-1})'(x) = 2\left(\frac{1+x}{x-1}\right)\left(\frac{-2}{(x-1)^{2}}\right)$$

$$\Rightarrow (f^{-1})'(2) = -12$$ ✓

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$$\frac{f(0) - f(-1)}{1} = -11 \Rightarrow 1 - (-8a+8) = -11$$

$$\Rightarrow a = \frac{1}{2}$$

$$f'(x) = 3(x^{2}+1)^{2}(2x)(ax+1) + (x^{2}+1)^{3}a$$

$$\Rightarrow f'(-1) = -8$$

✓

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$$\frac{\sin\frac{\pi}{2}\cos\pi - \sin\frac{\pi}{2}\cos\frac{\pi}{2}}{\frac{\pi}{4}} = -\frac{4}{\pi}$$

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$$\frac{\sin^4\frac{\pi}{2} - \cos^2\frac{\pi}{2} - \sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4}}{\frac{\pi}{4}} = \frac{4}{\pi}$$

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$$\frac{ax-1}{3x+1} = \frac{x}{7} + \frac{5}{7} \longrightarrow 7ax - 7 = 3x^2 + x + 15x + 5$$

$$3x^2 + x(16-7a) + 12 = 0 \xrightarrow{\Delta = 0} (16-7a)^2 - 12^2 = (16-7a-12)(16-7a+12)$$

$a = \frac{4}{7}$, $a = 4$

$a = 4 \longrightarrow 3x^2 - 12x + 12 \longrightarrow x = 2$ ✓

$a = \frac{4}{7} \longrightarrow 3x^2 + 12x + 12 \longrightarrow x = -2$ × ناحیه اول

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$$\lim_{x\to 0} \frac{\sin^n(x^2) \times 2x \sin^{n-1}(x^2) \times \cos x^2}{\left(2\sin^2 \frac{x}{2}\right)^m} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0} \frac{(x^2)^n \times n \times (x^2)^{n-1} \times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m} = \frac{(x)^{2n+2n-2+1} \times 2n}{(x)^{2m} \times \frac{1}{2}}$$

$$= \frac{2nx^{4n-1}}{\frac{1}{2^m} \times x^{2m}} = 2^{m+1} \times n \times x^{4n-2m-1}$$

پس $4n - 2m - 1 = 0 \longrightarrow n = \frac{2m+1}{4}$

لذا $2^{m+1} \times \frac{2m+1}{4} = 32\sqrt{2} \longrightarrow m = \frac{7}{2} , n = 2 \longrightarrow 2m + n = 9$

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$$y = -ax + 2 \longrightarrow f(4) = -4a + 2 \quad , \quad f'(4) = -a$$

$$-4a + 2 - a = -1 \longrightarrow -5a = -3 \longrightarrow a = +\frac{3}{5} \longrightarrow f'(4) = -\frac{3}{5} \stackrel{?}{=} -a$$