پارمیس نیک‌زاده ، دوازدهم دختر B — تکلیف شماره ۲۳

۱۸,۲۵

(۱)

$$g'\left(\frac{\pi}{۴}\right) = \left(-\sqrt{۲}\ \sin\frac{\pi}{۴} + ۲\left(۱+\tan^۲\frac{\pi}{۴}\right)\right) \times f'(۲)$$

$$\Rightarrow f'(۲)\times ۳ = \sqrt{۳} \qquad f'(۲) = \frac{\sqrt{۳}}{۳}$$

۲

(۲)

$$f(x) = \frac{\cancel{(۲+\cos x)}(\cos^۲ x + ۴ - ۲\cos x)}{(۲-\cos x)\cancel{(۲+\cos x)}}$$

۱,۵

$$f'\left(\frac{۷\pi}{۶}\right) = \frac{\left(۲\left(\frac{۱}{۲}\right)-۲\right)\left(۲-\frac{\sqrt{۳}}{۲}\right) - \left(-\frac{۱}{۲}\right)\left(\frac{۱۵}{۴}-\sqrt{۳}\right)}{\left(\frac{۴+\sqrt{۳}}{۲}\right)^۲} = \frac{۳}{۳۸+۱۶\sqrt{۳}}$$

$$g'\left(\frac{۷\pi}{۶}\right) = +\frac{\frac{۱}{۲}}{\frac{۱۹-۸\sqrt{۳}}{۴}} = \frac{۴}{۳۸-۱۶\sqrt{۳}} \qquad \text{حاصل} = \frac{-۵}{۳۸+۱۶\sqrt{۳}}$$

(۳)

$$x > \frac{۳}{۴} \rightarrow f' = \lim_{x\to\frac{۳}{۴}^+} \frac{(۴x-۳)\times\sqrt{ax} - ۰}{\left(x-\frac{۳}{۴}\right)} = ۴\sqrt{ax}$$

$$x < \frac{۳}{۴} \rightarrow f' = \lim_{x\to\frac{۳}{۴}^-} \frac{-(۴x-۳)\sqrt{ax} - ۰}{\left(x-\frac{۳}{۴}\right)} = -۴\sqrt{ax}$$

$$۴\sqrt{ax} - (-۴\sqrt{ax}) = ۸\sqrt{ax}$$

۲

$$۲\sqrt{۶} = ۸\sqrt{\frac{۳}{۴}a} = ۲\sqrt{۱۲a} \qquad ۱۲a = ۶ \qquad a = ۰/۵$$

(۴)

$$y = -ax + ۲$$

$$f(۴) = -۴a + ۲ \qquad f'(۴) = -a$$

$$f(۴) + f'(۴) = -۵a + ۲ = -۱ \qquad a = \frac{۳}{۵}$$

$$\Rightarrow f'(۴) = -a = -\frac{۳}{۵}$$

۲

$$y = \frac{x+5}{7} = \frac{ax-1}{3x+1} \Rightarrow 3x^2+16x+5 = 7ax-7$$ 5

$$\text{I}\quad 3x^2 + (16-7a)x + 12 = 0$$

$$y' = \frac{1}{7} = \frac{a+3}{(3x+1)^2} \rightarrow 9x^2+6x+1 = 7a+21$$

$$7a = 9x^2+6x-20 \quad \text{II}$$

$$\xrightarrow{\text{I},\text{II}} 3x^2 + (16-9x^2-6x+20)x + 12 = 0$$

$$9x^3 + 3x^2 - 36x - 12 = 0$$

$$3x^2(1+3x) - 12(1+3x) \rightarrow (1+3x)(3x^2-12) = 0$$

$$x = \frac{-1}{3}, +2, -2$$ در ناحیه $x = 2$

$$\text{II} \rightarrow 7a = 9(4) + 12 - 20 = 28 \qquad a = 4$$ 2

$$f(2) = 8 + 2a - b = 0 \qquad 2a - b = -8$$ 6

$$f'(2) = 3x^2 + a = 0 \longrightarrow 12 + a = 0$$

$$a = -12 \qquad b = -16 \qquad b - a = -4$$ 2

$$\frac{\sin^n(x^2) \times \frac{1}{x} \times \sin^n(x^2) \times (1+\cos x)^m}{x \quad (\sin^2 x)^m}$$ 7

$$= \frac{2^m \times \sin^{2n-2m}(0)}{x} = 2^{\frac{11}{2}}$$

$$2n - 2m = 0 \rightarrow m = n \qquad m = \frac{11}{2}$$

$$2m + n = 2\left(\frac{11}{2}\right) + \frac{11}{2} = 16/5$$ 1/5

$$\frac{\sqrt{x}+1}{\sqrt{x}-1}\times\frac{\sqrt{x}+1}{\sqrt{x}+1}=\frac{x+2\sqrt{x}+1}{x-1}$$ ⑧

$$f'(x)=\frac{\left(1+\frac{1}{\sqrt{x}}\right)(x-1)-(1)(x+2\sqrt{x}+1)}{(x-1)^2}$$

$$f'(2)=\left(1+\frac{\sqrt{2}}{2}\right)-(3+2\sqrt{2})=-2+\left(\frac{-3\sqrt{2}}{2}\right)$$

$$\left(f^{-1}(2)\right)'=\frac{1}{\frac{-3\sqrt{2}-4}{2}}=\frac{-2}{3\sqrt{2}+4}$$

۱,۵

آهنگ متوسط $=\dfrac{f(0)-f(-1)}{1}=\dfrac{1-(8-8a)}{1}=8a-7$ ⑨

$$8a-7=-11 \longrightarrow a=-0.5 \qquad x=-2a=1$$

$$f'(1)=(2x)\times 3\times(x^2+1)^2(0.5)+(-0.5)(8)$$

$$=24-4=20$$

۱,۲۵

آهنگ متوسط اولی $=\dfrac{\sin(\frac{\pi}{2})\cos\pi-\sin(\frac{\pi}{4})\cos(\frac{\pi}{2})}{\frac{\pi}{4}}$ ⑩

$$=\frac{-1}{\frac{\pi}{4}}$$

۲

آهنگ متوسط دومی $=\dfrac{\sin^4(\frac{\pi}{2})-\cos^4(\frac{\pi}{2})-\sin^4(\frac{\pi}{4})+\cos^4(\frac{\pi}{4})}{\frac{\pi}{4}}$

$$=\frac{1}{\frac{\pi}{4}}$$

$$\Rightarrow \frac{\frac{-1}{\frac{\pi}{4}}}{\frac{1}{\frac{\pi}{4}}}=(-1)$$

۶

$$f-2g\left(\frac{7\pi}{6}\right) = \frac{(2+\cancel{\cos x})(\cos^2 x-2\cos x+4)}{(2-\cos x)(2+\cancel{\cos x})} - \frac{8}{2-\cos x} = \frac{\overset{\cos x(\cos x-2)}{\cos^2 x-2\cos x}}{2-\cos x} = -\cos x$$

$$(f-2g)' = -(\cos x)' = +\sin x \to \sin\left(\frac{7\pi}{6}\right) = \boxed{-\frac{1}{2}}$$

۷

$$\lim_{x\to 0}\frac{\sin^n(x^2)\times 2x\sin^{n-1}(x^2)\times\cos x^2}{\left(2\sin^2\frac{x}{2}\right)^m} \xrightarrow{\text{هم‌ارزی}} \lim_{x\to 0}\frac{(x^2)^n\times n\times(x^2)^{n-1}\times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m} = \frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2m}\times\frac{1}{2}}$$

$$= \frac{2nx^{4n-1}}{\frac{1}{2^m}\times x^{2m}} = 2^{m+1}\times n\times x^{4n-2m-1} \to 4n-2m-1=0 \to n=\frac{2m+1}{4}$$

$$\to 2^{m+1}\times\frac{2m+1}{4} = 32\sqrt{2} \to m=\frac{7}{2},\ n=2 \to \boxed{2m+n=9}$$

۸

$$f^{-1}(2)=t \to f(t)=2 \to \sqrt{t}+1 = 2\sqrt{t}-2 \to \sqrt{t}=3 \to t=9$$

$${f^{-1}}'(2) = \frac{1}{f'(9)}\ ,\ f'(x) = \frac{-2}{(\sqrt{x}-1)^2}\times\frac{1}{2\sqrt{x}}$$

$$f'(9) = \frac{-2}{(2)^2}\times\frac{1}{6} = -\frac{1}{12} \to \boxed{{f^{-1}}'(2) = -12}$$

۹

$$\frac{f(0)-f(-1)}{0-(-1)} = \frac{1-8(-a+1)}{1} = 8a-7=-11 \to 8a=-4 \to \boxed{a=-\frac{1}{2}}$$

$$f'(x) = 3(x^2+1)^2(2x)(ax+1) + a(x^2+1)^3 \xrightarrow{-2a=1} f'(1) = 3(2)^2(2)\left(\frac{1}{2}\right) + -\frac{1}{2}(2)^3$$

$$f'(1) = 12-4 = \boxed{8}$$