[illegible] B-　　　　　　　۱۴٫۵　　　　　　　خورشید احمدی

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① $g'(x) = f'(\tan^2 x + \sqrt{2}\cos x)\cdot(2\tan x(1+\tan^2 x) - \sqrt{2}\sin x)$

$\Rightarrow g'(\frac{\pi}{4}) = f'(2)\cdot(3) = \sqrt{3} \Rightarrow f'(2) = \frac{\sqrt{3}}{3}$ ✓ ص

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② $f'(x) - 2g'(x) = (f-2g)'(x) \Rightarrow \frac{8+\cos^3 x}{4-\cos^2 x} - \frac{4(2+\cos x)}{(2-\cos x)(2+\cos x)}$

$= \frac{8+\cos^3 x - 8 - 4\cos x}{4-\cos^2 x} = \frac{\cos x(\cos^2 x - 4)}{4-\cos^2 x} = -\cos x$

$\Rightarrow f'(x) - 2g'(x) = \sin x \xrightarrow{x=\frac{7\pi}{4}} = \frac{-1}{2}$ ✓ ص

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③ $f(x)\begin{cases}(4x-3)\sqrt{ax} & x \geq \frac{3}{4}\\ (3-4x)\sqrt{ax} & x < \frac{3}{4}\end{cases} \Rightarrow f'(x)\begin{cases} y_1 = 4\sqrt{ax} + \frac{a(4x-3)}{2\sqrt{ax}} & x \geq \frac{3}{4}\\ y_2 = -y_1 & x < \frac{3}{4}\end{cases}$

$\xrightarrow{x=\frac{3}{4}} |y_2 - y_1| = 2\sqrt{6} \longrightarrow \cancel{2}\left(4\sqrt{ax} + \frac{a(4x-3)}{2\sqrt{ax}}\right) = \cancel{2}\sqrt{6}$

$\Rightarrow 2\sqrt{3a} + 0 = \sqrt{6} \Rightarrow a = \frac{1}{2}$ ✓ ص

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④ $\xrightarrow[f'(x) = -a]{y = 2 - ax} 2 - 4a - a = -1 \rightarrow \frac{3}{5} = a \Rightarrow f'(4) = \frac{3}{5}$ ✓ ص

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⑤ $y = \frac{5+x}{7} \rightarrow m_y = \frac{1}{7} = \frac{\begin{vmatrix} a & -1\\ 3 & 1\end{vmatrix}}{(3x+1)^2} = \frac{a+3}{(3x+1)^2}$ ۱٫۵

$7a + 21 = 9x^2 + 6x + 1 \Rightarrow 9x^2 + 6x - 20 - 7a = 0 \xrightarrow[\text{مماس}]{\Delta = 0}$

$36 + 36(20 + 7a) = 0 \Rightarrow 1 + 20 + 7a = 0 \Rightarrow \boxed{a = -3}$

$$\cancel{\text{x}} \qquad ۸ + ۲a - b = ۰ \Rightarrow \underbrace{b = ۸ + ۲a}$$

(۶)

$$f'(x) = ۰ \longrightarrow ۳x^۲ + a = ۰ \xrightarrow{x=۲} -۱۲ = a \ , \ b = -۱۶$$

$$\Rightarrow b - a = -۱۶ + ۱۲ = \boxed{-۴}$$

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(۷)

ب

$$\lim_{x\to ۰} \frac{\sin^n(x^۲) \times ۲x \sin^{n-1}(x^۲) \times \cos x^۲}{(۲\sin^۲\frac{x}{۲})^m} \overset{\text{هم‌ارزی}}{\leadsto} \lim_{x\to ۰} \frac{(x^۲)^n \times n \times (x^۲)^{n-1} \times ۲x}{(۲(\frac{x}{۲})^۲)^m} = \frac{(x)^{۲n+۲n-۲+۱} \times ۲n}{(x)^{۲m} \times \frac{۱}{۲}}$$

$$= \frac{۲nx^{۴n-1}}{\frac{1}{۲^m} \times x^{۲m}} = ۲^{m+1} \times n \times x^{۴n-۲m-1}$$

$$\to ۴n - ۲m - ۱ = ۰ \to n = \frac{۲m+۱}{۴}$$

$$\to ۲^{m+1} \times \frac{۲m+۱}{۴} = ۳۲\sqrt{۲} \longrightarrow m = \frac{۷}{۲} \ , \ n = ۲ \to ۲m + n = ۹$$

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(۸)

$$f'(x) = \frac{1}{(f^{-1})'(x)} \Rightarrow f'(x) = \frac{-۱}{\sqrt{x}} \ , \ f(x) = ۲ \Rightarrow x = ۹$$

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$$f'(۹) = -\frac{۱}{۳} \Rightarrow (f^{-1})'(۲) = \boxed{-۳}$$

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(۹)

$$۱ - ۸(۱-a) = -۱۱ \Rightarrow a = -\frac{۱}{۲}$$

$$\xrightarrow{x=۱} f'(x) = ۳(۲x)(x^۲+۱)^۲(ax+۱) + a(x^۲+۱)^۳$$

$$= ۶ \times ۴ \times \frac{۱}{۲} - ۴ = \boxed{۸}$$

ب

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(۱۰)

۲٫۵

$$\text{آهنگ متوسط} = \frac{-۱ - ۰}{\frac{\pi}{۴}} = \frac{-۴}{\pi} \ , \ y = -\cos ۲x \leadsto y' = ۲\sin ۲x$$

$$\xrightarrow{x = \frac{۳\pi}{۸}} ۲\sin\frac{۳\pi}{۴} = \sqrt{۲} \Rightarrow \frac{\frac{-۴}{\pi}}{\sqrt{۲}} = \frac{-۴}{\pi\sqrt{۲}}$$

۵

$$\frac{ax-1}{3x+1}=\frac{x}{7}+\frac{5}{7}\longrightarrow 7ax-7=3x^2+x+15x+5$$

$$3x^2+x(16-7a)+12=0\xrightarrow{\Delta=0}(16-7a)^2-12^2=(\underbrace{16-7a-12}_{a=\frac{4}{7}})(\underbrace{16-7a+12}_{a=4})$$

$$a=4\longrightarrow 3x^2-12x+12\longrightarrow x=2\ \checkmark$$

$$a=\frac{4}{7}\longrightarrow 3x^2+12x+12\longrightarrow x=-2$$ ناحیه اول ×

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$$f^{-1}(2)=t\rightarrow f(t)=2\longrightarrow \sqrt{t}+1=2\sqrt{t}-2\rightarrow\sqrt{t}=3\rightarrow t=9$$

$$(f^{-1})'(2)=\frac{1}{f'(9)}\quad,\quad f'(x)=\frac{-3}{(\sqrt{x}-1)^2}\times\frac{1}{2\sqrt{x}}$$

$$f'(9)=\frac{-3}{(2)^2}\times\frac{1}{6}=-\frac{1}{12}\longrightarrow (f^{-1})'(2)=-12$$

۱۰

$$x=\frac{\pi}{2}\longrightarrow \sin\frac{\pi}{4}\cos\frac{\pi}{2}=0$$

$$x=\frac{\pi}{2}\longrightarrow \sin\frac{\pi}{2}\cos\pi=-1$$

$$\longrightarrow\frac{-1-0}{\frac{\pi}{2}}=\overset{1}{\boxed{-\frac{4}{\pi}=\frac{\Delta y}{\Delta x}}}$$

$$\sin^4x-\cos^4x=(\sin^2x\overset{1}{\cancel{+}}\cos^2x)\overset{-\cos 2x}{(\sin^2x\cancel{-}\cos^2x)}=-\cos 2x$$

$$x=\frac{\pi}{2}\rightarrow -\cos\frac{\pi}{2}=0$$

$$x=\frac{\pi}{2}\rightarrow -\cos\pi=1$$

$$\longrightarrow\frac{1-0}{\frac{\pi}{2}}=\overset{2}{\boxed{\frac{4}{\pi}=\frac{\Delta y}{\Delta x}}}\longrightarrow\frac{-\frac{4}{\pi}}{\frac{4}{\pi}}=-1$$