نام و نام خانوادگی ...دارا رام میرزا ابتهاان... پاسخنامه تشریحی تکلیف شماره ۲۳ کلاس دوازدهم دختر B

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$$g'(x) = (\tan 2x + \sqrt{2}\cos x)' \cdot f'(\tan^2 x + \sqrt{2}\cos x)$$

$$g'(x) = \left[2\tan x \cdot (1+\tan^2 x) + (-\sqrt{2}\sin x)\right] \cdot f'(\tan^2 x + \sqrt{2}\cos x) = \sqrt{3}$$

$$\xrightarrow{x=\frac{\pi}{4}} \left[2\cdot(1+1) + (-1)\right] \cdot f'(2) = \sqrt{3} \Longrightarrow f'(2) = \frac{\sqrt{3}}{3}$$

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$$(f-2g)'\left(\tfrac{7\pi}{6}\right) = \frac{\overbrace{(4-4\cos x+\cos^2 x)}^{-\cos x\,\cancel{(-\cos x+4)}} \cancel{-4}}{\cancel{(2-\cos x)}} \longrightarrow (-\cos x)'_{\left(\frac{7\pi}{6}\right)} = \sin\left(\tfrac{7\pi}{6}\right) = -\frac{1}{2}$$

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$$f'(x) \xrightarrow{x\to\left(\frac{3}{4}\right)^-} -(4x-3)\sqrt{ax} \to -4\sqrt{ax} \xrightarrow{x=\frac{3}{4}} f'_-\left(\tfrac{3}{4}\right) = -2\sqrt{3a}$$

$$f'(x) \xrightarrow{x\to\left(\frac{3}{4}\right)^+} (4x-3)\sqrt{ax} \to 4\sqrt{ax} \xrightarrow{x=\frac{3}{4}} f'_+\left(\tfrac{3}{4}\right) = 2\sqrt{3a}$$

$$2(2\sqrt{3a}) = 2\sqrt{6} \to 4\sqrt{3a} = 2\sqrt{6} \to 2\sqrt{12a} = 2\sqrt{6} \to 12a = 6$$

$$a = \frac{1}{2}$$

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$$\left.\begin{aligned} f(4) &= -4a+2 \\ f'(4) &= -a \end{aligned}\right\} \xrightarrow{f(4)+f'(4)=1} -4a+2-a=1$$

$$-5a = -1 \to \boxed{a=\frac{1}{5}}$$

$$y = -ax+2$$

$$\xrightarrow{f'(4)} \boxed{-\frac{1}{5}}$$

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معادله برخورد $\longrightarrow$ ریشه مضاعف $\longrightarrow$ $\frac{ax-1}{3x+1} = \frac{5+x}{7} \to 7ax - 7 = 3x^2+16x+5$

$$3x^2 + (16-7a)x + 12 = 0$$

$$\Delta = 0 \to (16-7a)^2 - 4\times 3\times 12 = 0 \longrightarrow \underbrace{a = \frac{4}{7}}_{\text{غیرقابل} \times} \vee a = 4 \longrightarrow \text{ق ق}$$

$F(2)=0 \longrightarrow f(x) = x^3+ax-b \longrightarrow 8+2a-b=0 \longrightarrow \underbrace{2a-b=-8}$

$F'(2)=0 \longrightarrow 12+a=0 \longrightarrow a=-12$ $\qquad -24-b=-8$

$F'(x)=3x^2+a$ $\qquad b=-16$ ۶

$b-a=-16-(-12)=-4$

$\lim\limits_{x\to 0} \dfrac{F(x).F'(x)}{(1-\cos x)^m} = \lim\limits_{x\to 0} \dfrac{\overbrace{(\sin^n(x^2))}^{x^{2n} \leftarrow \text{هم ارزی}} \cdot \overbrace{F'(x)}^{2n\,x^{2n-1}}}{\underbrace{(1-\cos x)^m}} = \dfrac{x^{2n}.F'(x)}{\frac{x^2}{2}}$

هم ارزی $\dfrac{x^{2m}}{2^m}$ $\qquad 2m+n=9$ ۷

$\longrightarrow \lim\limits_{x\to 0} \dfrac{2^m \times 2n \times x^{4n-1}}{x^{2m}} = \dfrac{4n.x^{4n-1}\times 2^m}{x^{2m}} = 32\sqrt{2} \longrightarrow \dfrac{x^{4n-1}}{x^{2m}}$ باید یک شود $\dfrac{4n-1}{2}=m$ $\qquad n=\dfrac{2m+1}{4}$

$8n\times 2^m = \cancel{32}^{16}\sqrt{2} \longrightarrow \left(\dfrac{2m+1}{4}\right)\times 2^m = 16\sqrt{2} \longrightarrow (2m+1)\,2^m = 64\sqrt{2} \longrightarrow m=3/5$

$m(2^{m+1})+2^m \qquad \longrightarrow n=2$

$y=\dfrac{\sqrt{x}+1}{\sqrt{x}-1}$

نقطه تماس $(2,9)$

$F^{-1\prime}(2) = \bigcirc \longrightarrow F(\bigcirc) = 2 \longrightarrow \dfrac{\sqrt{x}+1}{\sqrt{x}-1}=2 \longrightarrow x=9$ ۸

$F^{-1}(x)$ مشتق $\longrightarrow \dfrac{1}{F'(x)} = \dfrac{1}{F'(9)}$ $\qquad F'(x) = \dfrac{-2}{(\sqrt{x}-1)^2}\times\dfrac{1}{2\sqrt{x}} \xrightarrow{x=9} F'(9)=-\dfrac{1}{12} \longrightarrow \dfrac{1}{F'(9)}=-12$

$\dfrac{F(0)-F(-1)}{0-(-1)} = \dfrac{1-8(1-a)}{1} = 8a-7=-11 \longrightarrow a=-\dfrac{1}{2} \longrightarrow -2a=1$

تغییر لحظه‌ای در $x=1$ $\longrightarrow F'(x) = 3(x^2+1)^2(2x)\left(-\frac{1}{2}x+1\right) + \left(-\frac{1}{2}\right)(x^2+1)^3 \xrightarrow{x=1} F'(1)=8$ ۹

$\dfrac{\text{آهنگ متوسط تابع } F(x)=\sin x\cos 2x}{\text{آهنگ متوسط تابع } g(x)=\sin^4 x-\cos^4 x} = \dfrac{\dfrac{F(\frac{\pi}{2})-F(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}}{\dfrac{g(\frac{\pi}{2})-g(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}} = \dfrac{\dfrac{\overbrace{\sin\frac{\pi}{2}}^{1}\overbrace{\cos\pi}^{-1}-\overbrace{\sin\frac{\pi}{4}}^{\frac{\sqrt{2}}{2}}\overbrace{\cos\frac{\pi}{2}}^{0}}{\cancel{\frac{\pi}{2}-\frac{\pi}{4}}}}{\dfrac{\underbrace{(\sin^4\frac{\pi}{2}-\cos^4\frac{\pi}{2})}_{1}-\underbrace{(\sin^4\frac{\pi}{4}-\cos^4\frac{\pi}{4})}_{0}}{\cancel{\frac{\pi}{2}-\frac{\pi}{4}}}} = -1$ ۱۰