نام و نام خانوادگی: دل‌آرام میرزا بابائیان — پاسخنامه تشریحی تکلیف شماره ۲۲ — کلاس: دوازدهم دختر B

۱۹/۵ آفرین

۱	$g'(x) = (\tan 2x + \sqrt{2}\cos x)' \cdot f'(\tan^2 x + \sqrt{2}\cos x)$ $g'(x) = [2\tan x \cdot (1+\tan^2 x) + (-\sqrt{2}\sin x)] \cdot f'(\tan^2 x + \sqrt{2}\cos x) = \sqrt{3}$ $\xrightarrow{x=\frac{\pi}{4}} [2\cdot(1+1) + (-1)] \cdot f'(2) = \sqrt{3} \Longrightarrow f'(2) = \frac{\sqrt{3}}{3}$ ۲
۲	$-\cos x \cancel{(-\cos x + 2)}$ $(f-2g)'(\frac{7\pi}{6}) = \frac{(4-4\cos x + \cos^2 x) \cancel{-4}}{\cancel{(2-\cos x)}} \longrightarrow (-\cos x)'_{(\frac{7\pi}{6})} = \sin(\frac{7\pi}{6}) = -\frac{1}{2}$ ۲
۳	$f'(x) \xrightarrow{x\to(\frac{3}{4})^-} -(4x-3)\sqrt{ax} \to -4\sqrt{ax} \xrightarrow{x=\frac{3}{4}} f'_-(\frac{3}{4}) = -2\sqrt{3a}$ $f'(x) \xrightarrow{x\to(\frac{3}{4})^+} (4x-3)\sqrt{ax} \to 4\sqrt{ax} \xrightarrow{x=\frac{3}{4}} f'_+(\frac{3}{4}) = 2\sqrt{3a}$ $2(2\sqrt{3a}) = 2\sqrt{6} \to 4\sqrt{3a} = 2\sqrt{6} \to 2\sqrt{12a} = 2\sqrt{6} \to 12a = 6$ $a = \frac{1}{2}$ ۲
۴	$f(4) = -4a+2$, $f'(4) = -a$ $\xrightarrow{f(4)+f'(4)=1} -4a+2-a=1$ $-5a=-1 \to \boxed{a=\frac{1}{5}}$ $y = -ax+2$ $\xrightarrow{f'(4)} \boxed{-\frac{1}{5}}$ ۱,۵
۵	معادله برخورد ⟵ ریشه مضاعف $\frac{ax-1}{3x+1} = \frac{5+x}{7} \to 7ax - 7 = 3x^2+16x+5$ $3x^2 + (16-7a)x + 12 = 0$ $\Delta = 0 \to (16-7a)^2 - 4\times 3\times 12 = 0 \to \underbrace{a = \frac{4}{7}}_{\text{ناصحیح } \times}$ یا $a = 4 \longrightarrow$ قق ۲

6

$F(2)=0 \longrightarrow f(x) = x^3+ax-b \longrightarrow 8+2a-b=0 \longrightarrow \underbrace{2a-b=-8}$

$F'(2)=0 \longrightarrow 12+a=0 \longrightarrow a=-12$ $\qquad -24-b=-8$

$F'(x)=3x^2+a$ $\qquad b=-16$

$b-a=-16-(-12)=-4$ ✓ 2

7

$$\lim_{x\to 0}\frac{F(x).F'(x)}{(1-\cos x)^m}=\lim_{x\to 0}\frac{\overbrace{(\sin^n(x^2))}^{x^{2n}\xleftarrow{\text{هم ارزی}}}\cdot F'(x)}{\underbrace{(1-\cos x)^m}}=\frac{x^{2n}.\overbrace{F'(x)}^{2n\,x^{2n-1}}}{\left(\frac{x^2}{2}\right)^m}$$ 2

$\frac{x^{2m}}{2^m}$ هم ارزی

آخرین این ضریب سوال نقص بودن

$2m+n=9$

$$\longrightarrow \lim_{x\to 0}\frac{2^m\times 2n\times x^{4n-1}}{x^{2m}}=\frac{4n.x^{4n-1}\times 2^m}{x^{2m}}=32\sqrt2 \longrightarrow \frac{x^{4n-1}}{x^{2m}}\xrightarrow[\text{یک شود}]{\text{باید}}\frac{4n-1}{2}=m$$ ✓

$n=\frac{2m+1}{4}$

$8n\times 2^m=\cancel{32}^{16}\sqrt2 \longrightarrow \left(\frac{2m+1}{4}\right)\times 2^m=16\sqrt2 \longrightarrow (2m+1)\,2^m=\frac{64\sqrt2}{2^{\frac{3}{2}}} \longrightarrow m=2/5$

$m(2^{m+1})+2^m \qquad \longrightarrow n=2$

8

$y=\frac{\sqrt x+1}{\sqrt x-1}$ $\quad$ ~~$F'(x) \to y\sqrt x-y=\sqrt x+1 \to (y-1)\sqrt x=1+y \to \sqrt x=\frac{1+y}{y-1}$~~

~~$x=\left(\frac{1+y}{y-1}\right)^2$ مشتق $\frac{(1)(x-1)^2-(1)(1+x)^2}{(x-1)^2}\xrightarrow{x=2}(1)(1)-(1)(9)$~~

~~$y=\left(\frac{1+x}{x-1}\right)^2$~~

نقطه مماس $(2,9)$ 2

$F^{-1}(2)=\bigcirc \longrightarrow F(0)=2\longrightarrow \frac{\sqrt x+1}{\sqrt x-1}=2\longrightarrow x=9$ ✓

$F^{-1}(x)$ مشتق $\longrightarrow \frac{1}{F'(x)}=\frac{1}{F'(9)}$ $\qquad F'(x)=\frac{-2}{(\sqrt x-1)^2}\times\frac{1}{2\sqrt x}\xrightarrow{x=9}F'(9)=-\frac{1}{12}\longrightarrow\frac{1}{F'(9)}=-12$

9

$$\frac{F(0)-F(-1)}{0-(-1)}=\frac{1-8(1-a)}{1}=8a-7=-11\longrightarrow a=-\frac{1}{2}\longrightarrow -2a=1$$

تغییر لحظه‌ای در $x=1$ $\longrightarrow F'(x)=3(x^2+1)^2(2x)\left(-\frac12 x+1\right)+\left(-\frac12\right)(x^2+1)^3\xrightarrow{x=1}F'(1)=8$ ✓ 2

10

$$\frac{\substack{\text{آهنگ متوسط تابع}\\ F(x)=\sin x\cos^3 x}}{\substack{\text{آهنگ متوسط تابع}\\ g(x)=\sin^4 x-\cos^4 x}}=\frac{\dfrac{F(\frac{\pi}{2})-F(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}}{\dfrac{g(\frac{\pi}{2})-g(\frac{\pi}{4})}{\frac{\pi}{2}-\frac{\pi}{4}}}=\frac{\dfrac{\overbrace{\sin\frac{\pi}{2}}^{1}\,\overbrace{\cos\pi}^{-1}-\overbrace{\sin\frac{\pi}{4}}^{\frac{\sqrt2}{2}}\,\overbrace{\cos\frac{\pi}{2}}^{0}}{\cancel{\frac{\pi}{2}-\frac{\pi}{4}}}}{\dfrac{\underbrace{(\sin^4\frac{\pi}{2}-\cos^4\frac{\pi}{2})}_{1}-\underbrace{(\sin^4\frac{\pi}{4}-\cos^4\frac{\pi}{4})}_{0}}{\cancel{\frac{\pi}{2}-\frac{\pi}{4}}}}=-1$$ ✓ 2

۴

$y = -ax + ۲ \longrightarrow f(۴) = -۴a + ۲$ و $f'(۴) = -a$

$-۴a + ۲ - a = -۱ \longrightarrow -۵a = -۳ \longrightarrow a = +\frac{۳}{۵} \longrightarrow f'(۴) = -\frac{۳}{۵} = -a$