

سوال ۱)

$$g(x) = f(\tan^2 x + \sqrt{2} \cos x) \xrightarrow{\text{مشتق}} g'(x) = (2(1 + \tan^2 x)(\tan x) - \sqrt{2} \sin x) f'(\tan^2 x + \sqrt{2} \cos x) \xrightarrow{g(\frac{\pi}{4}) = \sqrt{3}, x = \frac{\pi}{4}} \\ g'(\frac{\pi}{4}) = (2 \times 2 \times 1 - 1) f'(2) \rightarrow \sqrt{3} = 2 f'(2) \rightarrow f'(2) = \frac{\sqrt{3}}{2}$$

سوال ۲)

$$f(x) = \frac{1 + \cos^2 x}{x - \cos^2 x} \rightarrow f'(x) = \frac{(-2 \sin x \cos x)(x - \cos^2 x) - (1 + \cos^2 x)(-2 \sin x \cos x)}{(x - \cos^2 x)^2} \rightarrow f'(\frac{\sqrt{\pi}}{4}) = \frac{(-2 \times (\frac{1}{\sqrt{2}}) \times \frac{\sqrt{2}}{2})(\frac{\sqrt{\pi}}{4} - \frac{\sqrt{2}}{2}) - (1 + \frac{1}{2})(-\frac{\sqrt{2}}{2})(\frac{1}{\sqrt{2}})}{(\frac{\sqrt{\pi}}{4} - \frac{\sqrt{2}}{2})^2} \\ = \frac{135 - 128\sqrt{3}}{149 \times 2} \\ g(x) = \frac{2}{x - \cos x} \rightarrow g'(x) = \frac{-2 \sin x}{(x - \cos x)^2} \rightarrow g'(\frac{\sqrt{\pi}}{4}) = \frac{-2(\frac{1}{\sqrt{2}})}{\frac{19 + 18\sqrt{2}}{4}} = \frac{4}{19 + 18\sqrt{2}} = \frac{4(19 - 18\sqrt{2})}{149} \rightarrow 2g'(\frac{\sqrt{\pi}}{4}) = \frac{152 - 144\sqrt{2}}{149}$$

$$f'(\frac{\sqrt{\pi}}{4}) - 2g'(\frac{\sqrt{\pi}}{4}) = \frac{135 - 128\sqrt{3}}{149 \times 2} - \frac{152 - 144\sqrt{2}}{149} = \frac{135 - 128\sqrt{3} - 304 + 144\sqrt{2}}{149 \times 2} = \frac{-149}{149 \times 2} = -\frac{1}{2}$$

سوال ۳)

$$f(x) = |4x - 3| \sqrt{ax} \quad f'(\frac{3}{4}) = 4\sqrt{ax} + \frac{a}{2\sqrt{ax}}(4x - 3) = 4\sqrt{\frac{3a}{4}} \quad f'(\frac{3}{4}) = -4\sqrt{ax} + \frac{a}{2\sqrt{ax}}(3 - 4x) = -4\sqrt{\frac{3a}{4}} \\ f'(\frac{3}{4}) - f'(\frac{3}{4}) = 4\sqrt{\frac{3a}{4}} + 4\sqrt{\frac{3a}{4}} = 8\sqrt{\frac{3a}{4}} = 4\sqrt{3a} = 2\sqrt{4} \rightarrow \sqrt{4a} = \sqrt{2} \rightarrow a = \frac{1}{2}$$

سوال ۴)

$$y = -ax + 2 \quad f(x) + f'(x) = -1 \rightarrow -4a + 2 - a = -1 \rightarrow 5a = 3 \rightarrow a = \frac{3}{5} \rightarrow f'(x) = -a = -\frac{3}{5}$$

سوال ۵)

$$y = \frac{1}{\sqrt{x}} x + \frac{5}{\sqrt{x}} \quad y = \frac{ax - 1}{3x + 1} \rightarrow \frac{x + 5}{\sqrt{x}} = \frac{ax - 1}{3x + 1} \Rightarrow 3x^2 + 14x + 5 = 3ax - 1 \rightarrow 3x^2 + x(14 - 3a) + 6 = 0 \rightarrow \Delta = 0$$

$$\rightarrow 224 - 224a + 49a^2 - 144 = 0 \rightarrow 49a^2 - 224a + 112 = 0 \rightarrow (7a - 28)(7a - 4) = 0 \rightarrow \begin{cases} a = \frac{4}{7} \\ a = 4 \end{cases}$$

$$a = \frac{4}{7} \xrightarrow{0} 3x^2 + 12x + 12 = 0 \rightarrow x = -2 \rightarrow x \neq 0 \quad a = 4 \xrightarrow{0} 3x^2 - 12x + 12 = 0 \rightarrow x = 2 \quad \frac{-b}{2a}$$

سوال ۶)

$$f(x) = x^3 + ax - b \rightarrow f'(x) = 3x^2 + a \rightarrow f'(2) = 12 + a = 0 \rightarrow a = -12 \rightarrow f(2) = 8 - 24 - b = 0 \rightarrow b = -14 \quad b - a = -14 + 12 = -2$$

سوال ۷)

$$f(x) = \sin^n(x) \quad \lim_{x \rightarrow 0} \frac{f(x) f'(x)}{(1 - \cos x)^m} = 32\sqrt{2} \quad f'(x) = nx \cos(x) \sin^{n-1}(x)$$

سوال ۸)

$$f(x) = \frac{\sqrt{x} + 1}{\sqrt{x} - 1} = \frac{\sqrt{x} - 1 + 2}{\sqrt{x} - 1} = 1 + \frac{2}{\sqrt{x} - 1} \xrightarrow{f'(x) = f(x)} f'(x) = \frac{-1}{(\sqrt{x} - 1)^2} \times \frac{1}{2\sqrt{x}} \rightarrow f'(2) = \frac{-1}{3 - 2\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{-1}{3\sqrt{2} - 4} = \frac{-3\sqrt{2} - 4}{2}$$

سوال ۹)

$$f(x) = (x^2 + 1)^3 (ax + 1) \quad [-1, 0] \quad \text{طول بازه} = 0 - (-1) = 1 \quad f(-1) = 1(-a + 1) = 1 - a \quad f(0) = 1$$

$$\text{میانگین تغییرات} = \frac{f(0) - f(-1)}{\text{طول بازه}} = \frac{1 - (1 - a)}{1} = a \rightarrow 11 \rightarrow 11a = -4 \rightarrow a = -\frac{4}{11} \rightarrow -2a = \frac{8}{11}$$

$$f'(x) = 3(x^2 + 1)^2 (2x) + (-1)(x^2 + 1)^3 \xrightarrow{\text{میانگین تغییرات}} f'(-1) = 3(2)(4)(\frac{1}{2}) - 4 = 10$$

$f(x) = \sin x \cos 2x \quad \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ طول بازه = $\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$ $f\left(\frac{\pi}{4}\right) = -1$ $f\left(\frac{\pi}{2}\right) = 0$ $\sqrt{\text{اوسط متوسط تغییر}} = \frac{-1-0}{\frac{\pi}{4}} = -\frac{4}{\pi}$

$g(x) = \sin^2 x - \cos^2 x = -\cos 2x$ طول بازه = $\frac{\pi}{4}$ $g\left(\frac{\pi}{4}\right) = 1$ $g\left(\frac{\pi}{2}\right) = 0$ $\sqrt{\text{اوسط متوسط تغییر}} = \frac{1-0}{\frac{\pi}{4}} = \frac{4}{\pi}$

$\left. \begin{array}{l} -\frac{4}{\pi} \\ \frac{4}{\pi} \end{array} \right\} \frac{-\frac{4}{\pi}}{\frac{4}{\pi}} = -1$

سوال ۱۵)