

1. $f(x) = (x+1)^2 + (3x-1)^2 + mx^2 + nx + mn = 10x^2 - 4x + 2 + mx^2 + nx + mn$ چون تابع نایب با درجه 2 برابر باشد.

$m = -10, n = 4$

$g(x) = \{(4, m), (n, p+1), (p+2, -1), (-9, p+k)\}$

$(4, -10), (4, p+1) \rightarrow p+1 = -10 \Rightarrow p = -11$

$m+n+p+k = -10+4-9+8 = -7$

$p+2 = -9$

$p+k = -1 \Rightarrow k = 8$

الف) $\frac{x-2}{x^2-2x^2-8} \geq 0$ $\frac{-2}{-} + \frac{+2}{+} +$
 $x-2=0 \Rightarrow x=2$
 $x^2-2x^2-8=0 \Rightarrow (x^2-4)(x^2+2)=0 \Rightarrow x=\pm 2$
 $D_f = (-2, +\infty) - \{2\}$

ب) $\frac{2x^2+1}{4-2[-x]} \geq 0 \Rightarrow 4-2[-x] \neq 0 \Rightarrow 4 \neq 2[-x] \Rightarrow [-x] \neq 2 \Rightarrow -x \notin [2, 3]$

$x \notin (-3, 2] \Rightarrow D_f = \mathbb{R} - (-3, 2]$

الف) $\frac{\sqrt{x(x^2-1)}}{\sqrt{|x|+x}} =$ I) $x(x^2-1) \geq 0 \Rightarrow \frac{-1}{-} + \frac{0}{+} - \frac{1}{+}$ III) $|x|+x \neq 0$
 II) $|x|+x \geq 0 \Rightarrow x \in [0, +\infty)$ $x \in (0, +\infty)$

$I \cap II \cap III = [1, +\infty) = D_f$

ب) $\frac{\sqrt{|x-2|-x^2}}{|x|-1}$ I) $|x-2|-x^2 \geq 0 \Rightarrow x-2 \geq x^2 \Rightarrow x^2-x+2 \leq 0$ $\Delta < 0$
 $-x+2 \leq x^2 \Rightarrow x^2+x-2 \geq 0$ $\frac{-2}{+} + \frac{1}{-} +$
 II) $|x|-1 \neq 0 \Rightarrow |x| \neq 1 \Rightarrow x \neq \pm 1$
 $D_f = (-\infty, 2], (1, +\infty)$

$||x-2|-1| = k$ $|x-2| = k+1 \Rightarrow |x-2| \geq 1$ $x-2 \geq 1 \Rightarrow x \geq 3 \Rightarrow x-2 = k+1 \Rightarrow x = k+3$
 $x-2 \leq -1 \Rightarrow x \leq 1 \Rightarrow -x+2 = k+1 \Rightarrow x = -k+1$
 $|x-2|-1 = \pm k$ $|x-2| = 1-k \rightarrow |x-2| \leq 1$ $1 \leq x-2 \leq 1 \Rightarrow 1 \leq x \leq 2 \Rightarrow -x+2 = 1-k \Rightarrow x = k+1$
 $1 \leq x \leq 3$ $2 \leq x \leq 3 \Rightarrow x-2 = 1-k \Rightarrow x = 3-k$

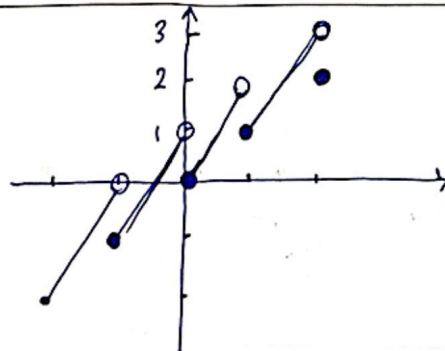
$[-2, -1) \Rightarrow 2x+2$

$[-1, 0) \Rightarrow 2x+1$

$[0, 1) \Rightarrow 2x$

$[1, 2) \Rightarrow 2x-1$

$2 \Rightarrow 2x-2$



$$\frac{x^2 - 4x + 3}{x+a} + b \Rightarrow \frac{(x-3)(x-1)}{x+a} + b = \frac{x^2 - 4x + 3}{x+y} \quad \text{چون تابع همان است}$$

$$\text{I) } \left. \begin{array}{l} a = -3 \\ b = 1 \end{array} \right\} \frac{(x-3)(x-1)}{x-3} + 1 \Rightarrow x-1+1=x \rightarrow a-b = -4$$

$$\text{II) } \left. \begin{array}{l} a = -1 \\ b = 3 \end{array} \right\} \frac{(x-3)(x-1)}{x-1} + 3 \Rightarrow x-3+3=x \rightarrow a-b = -4$$

$$\left. \begin{array}{l} -4-4 = -8 \end{array} \right\}$$

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$$g(x) = f(x) \Rightarrow x=2 \Rightarrow f(2)=4 \rightarrow x+C=4 \rightarrow C=2$$

$$x=1 \rightarrow g(1)=3 \quad f(1) = \frac{a+2}{1+b} = 3 \Rightarrow 3+3b = a+2 \rightarrow a = 3b+1$$

$$x=-1 \rightarrow g(-1)=1 \quad f(-1) = \frac{2-a}{-1+b} = 1 \Rightarrow 2-a = -1+b \quad \frac{a=3b+1}{\rightarrow 2-3b-1 = -1+b}$$

$$\left. \begin{array}{l} 4b=2 \Rightarrow b=\frac{1}{2} \\ a=\frac{5}{2} \end{array} \right\} \quad b+C = 2 + \frac{1}{2} = \frac{5}{2}$$

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$$D_f = 4x - 3a > 0 \Rightarrow 4x > 3a \Rightarrow x > \frac{3a}{4} \rightarrow \left[\frac{3a}{4}, +\infty \right) \quad \left[\frac{3a}{4}, +\infty \right) \cap \left(-\infty, \frac{b}{4} \right] = \{ \}$$

$$D_g = b - 4x > 0 \Rightarrow b > 4x \Rightarrow \frac{b}{4} > x \Rightarrow \left(-\infty, \frac{b}{4} \right]$$

$$\frac{3a}{4} = 1 \Rightarrow 3a = 4 \rightarrow a = \frac{4}{3} \quad \frac{b}{4} = 1 \Rightarrow b = 4 \quad 3a - \frac{b}{2} - k = 4 - 2 - 1 = 1$$

$$\left. \begin{array}{l} f(1) = \sqrt{4-4} + k - 1 \\ g(1) = \sqrt{4-4} + 3k \end{array} \right\} \quad 2k - 2 - 3k = k \Rightarrow k = -1$$

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$$D_f \cap D_g = [-1, 7] \Rightarrow D_g = 2x + 2 > 0 \Rightarrow 2x > -2 \Rightarrow x > -1$$

$$D_f = m - x > 0 \Rightarrow m > x \Rightarrow m = 7$$

$$\left. \begin{array}{l} f(3) - g(3) = 6\sqrt{2} \\ g(3) = \sqrt{8} \Rightarrow 2\sqrt{2} \end{array} \right\} \quad f(3) = 8\sqrt{2}$$

$$f(3) = \sqrt{7-3} + n = \sqrt{4} + n = 2 + n = 8\sqrt{2}$$

$$n = 8\sqrt{2} - 2$$

$$m + n = 8\sqrt{2} - 2 + 7 = 8\sqrt{2} + 5$$

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$$y = x - [x] = \frac{1}{2}$$

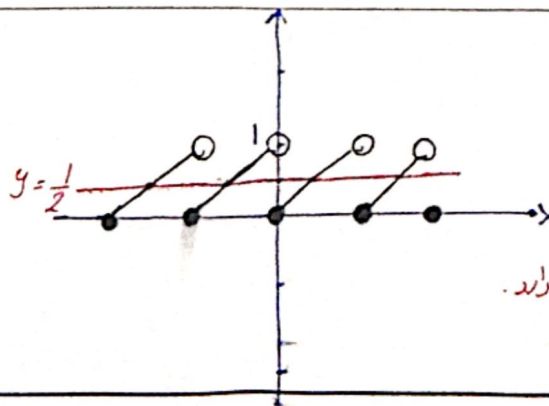
$$[-2, -1) : x + 2$$

$$[-1, 0) : x + 1$$

$$[0, 1) : x$$

$$[1, 2) : x - 1$$

$$2 = x - 2$$



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