

$$x^r + 2m + 1 + 9x^r + 1 - 4m \Rightarrow 10x^r - 2m + 2 + m x^r + n n + m n = f(m)$$

$$\Rightarrow x^r(10 + m) + m(n - 2) + m n = f(m)$$

تابع ثابت: $10 + m = 0 \sim m = -10$, $n - 2 = 0 \sim n = 2$

$$g(m) = \{(2, -10), (2, p+1), (\frac{p+2}{-9}, -1), (-9, p+2)\} \quad p+1 = -10 \Rightarrow p = -11$$

$$p + k = -1 \sim -11 + k = -1 \sim k = 10 \Rightarrow m + n + p + k = -10 + 2 - 11 + 10 = -9$$

الف) $\sqrt{\frac{x-2}{x^2-4x-1}} \sim \frac{x-2}{x^2-4x-1} = 0$
 $(x-2)(x+1) = 0$
 $x^2 - 4x - 1 = 0 \Rightarrow x = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$
 $D_f = (-2, +\infty)$
 $D_f = (-2, +\infty) - \{2\}$

ب) $f \neq 2[-m] \Rightarrow f \neq [-m] \Rightarrow f \leq -m < 3 \Rightarrow -2 \geq m > -3$

$$D_f = (-3, -2] \quad R - (-3, -2]$$

الف) $I: x(x^2-1) \geq 0 \Rightarrow \begin{cases} x=0 \\ x^2-1=0 \Rightarrow x=\pm 1 \end{cases}$
 $II: |x| + x > 0 \Rightarrow \begin{cases} x > 0 \\ x < 0 \end{cases} \Rightarrow (0, +\infty)$
 $I \cap II: [1, +\infty)$

ب) $|x-2| - x^2 \geq 0 \Rightarrow \begin{cases} x-2 \geq x^2 \Rightarrow ① \quad 0 \geq x^2 - x + 2 \\ x-2 \leq -x^2 \Rightarrow ② \quad x^2 + x - 2 \leq 0 \end{cases}$

①: $\Delta < 0 \quad x = \frac{1 \pm \sqrt{1-8}}{2} \Rightarrow$ ریشه ندارد
 ②: $(x+2)(x-1) = 0 \Rightarrow \frac{-2 \pm 1}{1 \pm 1} = [-2, 1]$

$$D_f = [-2, 1) - \{-1\}$$

①: $|x-2| - 1 \neq k \quad |x-2| \geq 1 \Rightarrow x \geq 3, x \leq 1$
 ②: $1 - |x-2| \neq k \quad |x-2| < 1 \Rightarrow 1 < x < 3$

$$D_f = R - \{ \text{ریشه کسری} \}$$

$\Rightarrow ①: |x-2| \neq k+1 \Rightarrow \begin{cases} x-2 \neq k+1 \Rightarrow x \neq k+3 \quad x \geq 2 \Rightarrow [2, +\infty) \\ x-2 \neq -(k+1) \Rightarrow x \neq 1-k \quad x \leq 2 \Rightarrow (-\infty, 2] \end{cases}$
 $D_{f①} = (-\infty, 2] \cup [3, +\infty)$

②: $|x-2| \neq 1-k \Rightarrow \begin{cases} x-2 \neq 1-k \Rightarrow x \neq 3-k \quad x \geq 2 \Rightarrow [2, 3) \\ x-2 \neq -(1-k) \Rightarrow x \neq 1+k \quad x < 2 \Rightarrow (1, 2) \end{cases}$
 $D_{f②} = (1, 3)$

$\Rightarrow \boxed{k=1}$

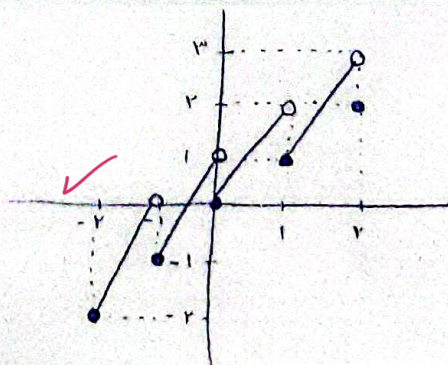
$$2m+2 \quad -2 \leq m < -1$$

$$2m+1 \quad -1 \leq m < 0$$

$$2m \quad 0 \leq m < 1$$

$$2m-1 \quad 1 \leq m < 2$$

$$2m-2=2 \quad m=2$$



$$\frac{x^r - rx + r}{x+a} = x-b \Rightarrow x^r - bx + ax - ab = x^r - rx + r \Rightarrow \begin{cases} a-b = -r \\ -ab = r \end{cases}$$

$$a = b-r \Rightarrow ab = -r \Rightarrow (b-r)b = -r$$

$$\Rightarrow b^r - rb + r = 0 \Rightarrow (b-1)(b-r) = 0 \Rightarrow \begin{cases} b=1 \Rightarrow a=b-r \\ b=r \end{cases} \begin{cases} a=-r \rightarrow b=1 \\ a=-1 \rightarrow b=r \end{cases}$$

$$\begin{cases} -r-1 = -r \\ -1-1 = -2 \\ -r-r = -2r \\ -1-r = -r-1 \end{cases} \quad \mu_{\text{مقدار}} = \begin{cases} -r, -1, -2r \end{cases} \quad \begin{aligned} a < -1 &\rightarrow x-r+b=x \rightarrow b=r \quad a-b < -r \\ a < -r &\rightarrow x-1+b=x \rightarrow b=1 \rightarrow a-b < -r \end{aligned}$$

$$f(x) = \begin{cases} \frac{(x^r-1)(x+r)}{(x^r-1)} = x+r & (x^r \neq 1) \\ \frac{ax+r}{x+b} & (x^r = 1) \end{cases} \quad x+r = x + \frac{r}{r} \Rightarrow C=r$$

$$x=1: \frac{a+r}{1+b} = r \Rightarrow r+rb = a+r \Rightarrow a = rb+1$$

$$x=-1: \frac{-a+r}{b-1} = -1 \Rightarrow b-1 = -a+r \xrightarrow{\text{جایگزینی}} b-1 = -rb-1+r$$

$$\Rightarrow rb = r \Rightarrow b = \frac{1}{r} \quad b+C = \frac{1}{r} + r$$

$$D_f = rx - ra \geq 0 \Rightarrow x \geq \frac{ra}{r} \quad D_g = b - rx \geq 0 \Rightarrow x \leq \frac{b}{r}$$

$$r \neq g = \{(1, k)\} \Rightarrow D_f \cap D_g = \{1\} \quad \frac{ra}{r} = \frac{b}{r}$$

$$f(1) = \sqrt{r-ra} + k-1 \quad g(1) = \sqrt{b-r} = \sqrt{ra-r} \Rightarrow a = \frac{r}{r} > b=r$$

$$r \neq g(1) = k \Rightarrow rk - r = k \Rightarrow k = r \Rightarrow ra - \frac{b}{r} - k = r - r - r = -r$$

$$rk - r - rk < k \rightarrow rk < -r \rightarrow k < -1$$

$$D_g \Rightarrow rx \geq -r \Rightarrow x \geq -1 \quad D_f = m - x \geq 0$$

$$D_{f \times g} = [-1, r] \quad \text{است. دایره های} \quad \Rightarrow D_f = (-\infty, r] \Rightarrow r \geq x \Rightarrow m = r$$

تابع f در $[-\infty, x_0]$ است پس فقط دایره های آن $(-\infty, x_0]$ ←
 $(x_0, +\infty)$ ←

$$\sqrt{\frac{r-x}{r}} + n - \sqrt{\frac{rx+r}{1}} = 4\sqrt{r}$$

$$\Rightarrow n = 4\sqrt{r} + \sqrt{\frac{rx+r}{1}} - r \Rightarrow n = 4\sqrt{r} - r \Rightarrow m+n = r + 4\sqrt{r} - r = 4\sqrt{r}$$

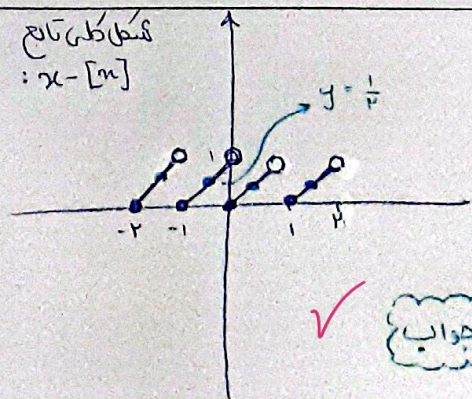
$$-r \leq x < -1 \quad \sim \quad x+r = \frac{1}{r}$$

$$-1 \leq x < 0 \quad \sim \quad x+1 = \frac{1}{r}$$

$$0 \leq x < 1 \quad \sim \quad x = \frac{1}{r}$$

$$1 \leq x < r \quad \sim \quad x-1 = \frac{1}{r}$$

$$x=r \quad \sim \quad \text{مقدار}$$



✓ جواب