دوازدهم ریاضی B

کیمیا امیک نیا

۱۹

① ②

$$f(x)=(x+1)^2+(3x-1)^2+mx^2+nx+mn$$

$$f(x)=\underbrace{x^2+2x+1+9x^2}_{10x^2}\overbrace{-6x+1}^{-4x}+mx^2+nx+mn=(10+m)x^2+(n-4)x+1+mn$$

$$\to 10+m=0\to m=-10$$
$$\to n-4=0\to n=4 \Big\} \to mn=-40$$

$$g(x)=\{(4,m),(n,P+1),(P+2,-1),(-9,P+K)\}=\{(4,-10),(4,P+1),(P+2,-1),(-9,P+K)\}$$

$$\to P+1=-10\to P=-11\to P+2=-9\to (P+2,-1)=(-9,-1)$$

$$\to P+K=-1\to -11+K=-1\to K=10$$

$$m+n+P+K=-10+4-11+10=\boxed{-7}$$

---

② الف)

$$x^4-2x^2-8\neq 0 \xrightarrow{x^2=t} t^2-2t-8=0 \to t=-2 \quad ,\quad t=4\to x^2=4\to x=\pm 2$$

$$y\geq 0$$

-2		2*
−	+	+

$$x-2=0\to x=2$$

$$D_y=(-2,+\infty)-\{2\}$$

ب)

$$4-2[-x]\neq 0 \to -2[-x]\neq -4\to [-x]\neq 2$$

$$[-x]=2\to 2\leq -x<3\to -3<x\leq -2$$

$$D_f=\mathbb{R}-(-3,-2]$$

---

③ ②

الف) $x(x^2-1)\geq 0$

-1		0		1	
−	+		−		+

①

$$x+|x|>0\to x+|x|\neq 0\to |x|\neq -x\to x>0 \quad ②$$

همواره نامنفی هست. فقط باید صفر نشود!

$$D_f=①\cap②=[1,+\infty)$$

ب)

$|x|-1 \neq 0 \to |x| \neq 1 \to x \neq \pm 1$ ①

$|x-2| - x^2 \geq 0$

$$\begin{array}{c|c} & 2 \\ \hline -x^2-x+2 \geq 0 & -x^2+x-2 \geq 0 \end{array}$$

$-1 \leq x \leq 2$ ②

$D_f = ① \cap ② = (-1,2] - \{1\}$ ✓

---

④ ①

$||x-2|-1| \neq k \to |x-2|-1 \neq k \to |x-2| \neq k+1$

$|x-2|-1 \neq -k \to |x-2| \neq -k+1$

$$\begin{cases} x-2 \neq -k+1 \\ x-2 \neq k+1 \\ x-2 \neq -k+1 \\ x-2 \neq k-1 \end{cases}$$

می‌توانن برابر باشن

نمی‌توانن برابر باشن

$k+1 = -k-1 \to k = -1$

$k+1 = -k+1 \to k = 0$ ✓

با رسم شکل راحت به جواب می‌رسیم!

$|x-2|-1=0 \to |x-2|=1$ ۲ جواب

---

⑤ ②

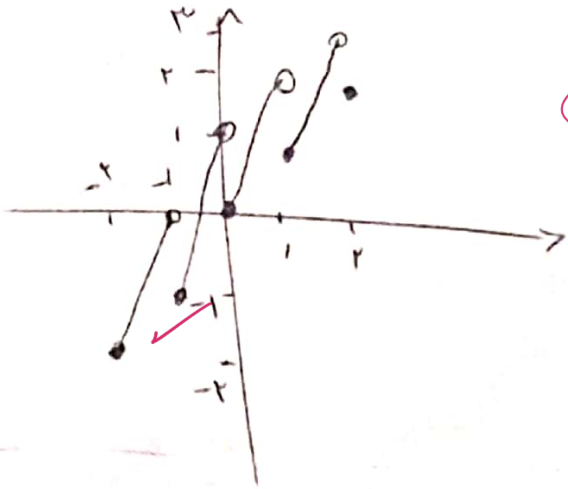
$-2 \leq x < -1 \to [x] = -2 \to y = 2x+2$

$-1 \leq x < 0 \to [x] = -1 \to y = 2x+1$

$0 \leq x < 1 \to [x] = 0 \to y = 2x$

$1 \leq x < 2 \to [x] = 1 \to y = 2x-1$

$x = 2 \to [x] = 2 \to y = 2$



---

⑥ ②

$$\frac{x^2-4x+3}{x+a} + b = x \to \frac{x^2-4x+3}{x+a} = (x-b)$$

$x^2-4x+3 = (x+a)(x-b) = x^2+(a-b)x-ab \to a-b=-4$ ✓

(7)

(2)

$$\frac{x^3-2x^2-x-2}{x^2-1} \underset{x\neq\pm1}{\Longrightarrow} \frac{(x+1)(x-1)(x+2)}{(x+1)(x-1)} = x+2$$

دامنه درسته

$$x+c = x+2 \rightarrow c=2$$

$$\frac{ax+2}{x+b} = x+2 \xrightarrow{x=1} \frac{a+2}{b+1} = 3 \rightarrow a+2 = 3b+3$$

$$x=\pm1 \qquad \xrightarrow{x=-1} \frac{-a+2}{b-1} = 1 \rightarrow -a+2 = b-1$$

$$\rightarrow a=\frac{5}{2} \qquad b=\frac{1}{2}$$

$$b+c = \frac{1}{2}+2 = \boxed{\frac{5}{2}}$$ ✓

$$2f-g\,(1) = k$$

(8) (2)

f و g مقدار $a=1$
دامنه‌ی مشترک دارند

$$\rightarrow D_f : x \geq \frac{3a}{4}$$

$$\rightarrow D_g = b \geq 4x \rightarrow x \leq \frac{b}{4} \quad \rightarrow \frac{3a}{4} = \frac{b}{4} \rightarrow 3a=b$$

$$\frac{3a}{4} = \frac{b}{4} = 1 \rightarrow b=4 \qquad a=\frac{4}{3}$$

$$2f-g(1) = 2\sqrt{4} + 2k-2 - \sqrt{4-4} + 3k = k$$

$$\rightarrow 2k=-2 \rightarrow k=-1$$

$$3a-\frac{b}{2}-k = 4-2+1 = \boxed{3}$$ ✓

$$D_{fog} = D_f \cap D_g = [-1,7]$$

(9) (2)

$$D_g = 3x+3 \geq 0 \rightarrow x \geq -1 \quad \rightarrow D_f : x \leq 7 \rightarrow m \geq x \rightarrow m=7$$

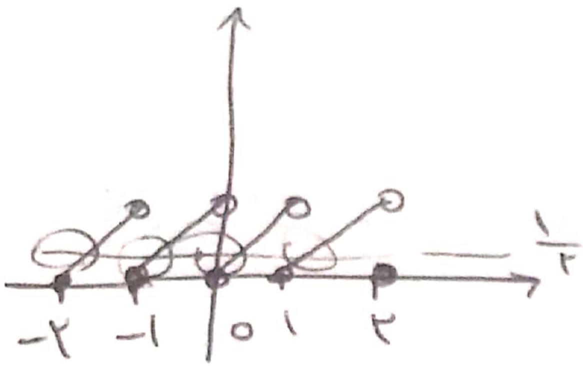
$$f-g\,(3) = 2+n-2\sqrt{2} = 6\sqrt{2} \rightarrow n = 8\sqrt{2}-2$$

$$m+n = 8\sqrt{2}-2+7 = \boxed{5+8\sqrt{2}}$$ ✓

۲ ۱۰

$y=(x-[x])$

$y=\frac{۱}{۲}$



$\longrightarrow$ ۴ جواب