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$$f(x) = x^2 + 2x + 1 + 9x^2 - 6x + 1 + mx^2 + nx + mn \xrightarrow{\text{جمع می‌کنیم}} f(x) = A \quad m = -10, n = 4 \rightarrow f(x) = mn + 2 \rightarrow$$

$$f(x) = -2A$$

اگر $g(x)$ به صورت $g(x) = (x, p+1), (p+1, -1), (-1, p+k), (-1, p+k)$ $\rightarrow p+1, -1 \rightarrow p, -11, -11+k, -1 \rightarrow k=10$

$$\Rightarrow m+n+p+k = -10+4-11+10 = -7 \quad \checkmark$$

الف) $y = \sqrt{\frac{x-2}{x^2-2x-2}} \rightarrow y = \sqrt{\frac{x-2}{(x-1)(x+2)}} \rightarrow y = \sqrt{\frac{x-2}{(x-1)(x+2)(x^2+2)}}$

$\omega = \frac{x-2}{x^2+2} = \frac{x-2}{x^2+2} \rightarrow \frac{-2}{x^2+2} + \frac{1}{x^2+2} + \frac{1}{x^2+2}$

$D_f = (-2, 2) \cup (2, +\infty)$

ب) $f(x) = \frac{x^2+1}{x^2-2} \rightarrow x^2-2 \neq 0 \rightarrow x^2 \neq 2 \rightarrow x \neq \pm\sqrt{2} \rightarrow x \neq (-\sqrt{2}, -\sqrt{2})$

$$\Rightarrow D_f = \mathbb{R} - (-\sqrt{2}, -\sqrt{2}) \quad \checkmark$$

الف) $f(x) = \frac{x(x-1)(x+1)}{\sqrt{x+1}}$: $\textcircled{1} x(x-1)(x+1) \geq 0 \rightarrow \frac{-1}{0} + \frac{1}{0} - \frac{1}{0} + \frac{1}{0} \rightarrow [-1, 0] \cup [1, +\infty) \textcircled{2} x+1 \geq 0 \rightarrow x \geq -1$

$$x > 0 \rightarrow \text{اشتباه} : D_f = [1, +\infty) \quad \checkmark$$

ب) $f(x) = \frac{\sqrt{|x-1|-x^2}}{|x|-1}$: $\textcircled{1} |x| \neq 1 \rightarrow x \neq \{\pm 1\}, \textcircled{2} |x-1|-x^2 \geq 0 \rightarrow x^2 - x + 1 \leq 0 \rightarrow (x-1/2)^2 \geq 3/4 \rightarrow x \leq -1/2 \text{ or } x \geq 3/2$

$$\Rightarrow D_f = [-2, 1) - \{-1\} \quad \checkmark$$

$|x-1|-1 = k : \textcircled{1} |x-1|-1 \leq k \rightarrow |x-1| \leq k+1 \rightarrow x \geq 2 : x-2 \leq k+1 \rightarrow x \leq k+3$
 $\hookrightarrow x < 2 : x-2 \geq -k-1 \rightarrow x \geq 1-k$

$\textcircled{2} |x-1|-1 = -k \rightarrow |x-1| = 1-k \rightarrow x \geq 2 : x-2 = 1-k \rightarrow x = 3-k$
 $\hookrightarrow x < 2 : x-2 = -1+k \rightarrow x = k+1$

$k+3 = 1-k \rightarrow k = -1 \quad \checkmark$

$k+1 = 3-k \rightarrow k = 1 \quad \checkmark$

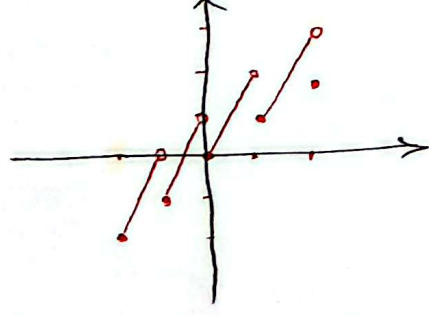
$y = 2x - [x] : x = [-2, -1) : 2x+2$

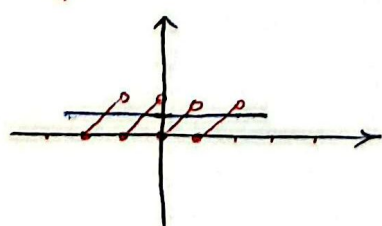
$x = [-1, 0) : 2x+1$

$x = [0, 1) : 2x$

$x = [1, 2) : 2x-1$

$x = 2 : 2x-2$



$\frac{x^2 - 4x + 4}{x+a}$, $x-b \rightarrow$: $\frac{x^2 - 4x + 4}{x+a}$ بر $x-b$ تقسیم می شود $\rightarrow$ $(x+a)(x-b) = (x-2)(x-1) \rightarrow \textcircled{1} a = -2, b = 1 \quad \textcircled{2} a = -1, b = 2 \rightarrow$ $\textcircled{1} \rightarrow a-b = -2-1 = -3$ $\textcircled{2} \rightarrow a-b = -1-2 = -3$ $\rightarrow a-b = -3$	۶
$f(0) = 2, g(0) = C \Rightarrow \underline{C=2}$ $g(1) = 2, f(1) = \frac{a+2}{b+1} \rightarrow 2b+2 = a+2 \rightarrow 2b-a = 0$ $g(-1) = 1, f(-1) = \frac{-a+2}{b-1} \rightarrow b-1 = -a+2 \rightarrow b+a = 1$ $\rightarrow 2b = 1 \rightarrow b = \frac{1}{2}$ $b+C = 2 + \frac{1}{2} = \underline{\frac{5}{2}}$	۷
$f: (-\infty, 1] \rightarrow \mathbb{R}$ و $g: [1, +\infty) \rightarrow \mathbb{R}$ $f(x) - f(a) > 0 \rightarrow f(x) > f(a) \rightarrow x > \frac{a}{2} \rightarrow \frac{a}{2} > 1 \rightarrow \underline{a > 2}$ $b - f(x) > 0 \rightarrow f(x) < b \rightarrow x < \frac{b}{2} \rightarrow \frac{b}{2} < 1 \rightarrow \underline{b < 2}$ $(f-g)(1) = 2(k+1) - 2k = k \rightarrow 2-k = k \rightarrow \underline{k=1}$ $2a - \frac{b}{2} = k = 2 - \frac{1}{2} = \underline{\frac{3}{2}}$	۸
$g(x) = 2x+2 \geq 0 \rightarrow x \geq -1 \rightarrow D_g = [-1, +\infty) \Rightarrow$ $D_f = (-\infty, 2] \rightarrow m-n \geq 0 \rightarrow x \leq m \Rightarrow \underline{m=2}$ $(f-g)(x) = \sqrt{2x-2} + x - \sqrt{2x+2} = 4\sqrt{2} \rightarrow 2-2\sqrt{2}+n = 4\sqrt{2} \rightarrow \underline{n = 8\sqrt{2}-2}$ $\Rightarrow m+n = 2 + 8\sqrt{2} - 2 = \underline{8\sqrt{2}}$	۹
$y = \frac{1}{x} \rightarrow y = \frac{1}{x-[x]} \rightarrow y = \frac{1}{x-[x]}$ $y = \frac{1}{x}$ خط و $x-[x]$ تابع پله ای $[-5, 2]$ 	۱۰