مسابقاتی / تکلیف شماره‌ی ۱۹

① $f(x)=2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right]$ $\qquad \left[\frac{a}{3}\right]=?$

$\lim_{x\to -2^+} f(x)=\lim_{x\to -2^-} f(x) \longrightarrow 2+0=4-a \to a=2$

$\left[\frac{2}{3}\right]=0$

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② $\lim_{x\to(\frac{\pi}{6})^-}[2\sin x-1]=[0^-]=-1$

$x<\frac{\pi}{6}\Longrightarrow \sin x<\frac{1}{2}\to 2\sin x<1\to 2\sin x-1<0$

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③ $\lim_{x\to -\frac{1}{2}}[8x^3-x]=\left[8(-\frac{1}{2})^3-(-\frac{1}{2})\right]=\left[-1+\frac{1}{2}\right]=\left[-\frac{1}{2}\right]=-1$

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④ $\lim_{x\to(-\frac{1}{2})^-}\frac{10x-5+\left[\frac{3}{x^2}\right]}{16x-\left[-\frac{2}{x^2}\right]}=\lim_{x\to(-\frac{1}{2})^-}\frac{10x\overbrace{-5+11}^{6}}{16x+8}=\frac{-5^-+6}{-8^-+8}=\frac{1}{0^-}=-\infty$

$x<-\frac{1}{2}\to x^2>\frac{1}{4}\to\frac{1}{x^2}<4\to\frac{3}{x^2}<12$

$x<-\frac{1}{2}\to x^2>\frac{1}{4}\to\frac{1}{x^2}<4\to\frac{2}{x^2}<8\to-\frac{2}{x^2}>-8$

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⑤ $\lim_{x\to 1^+}\frac{\sin^2\pi x}{[x]+\cos\pi x}=\lim_{x\to 1^+}\frac{\sin^2\pi x}{1+\cos\pi x}$ $\qquad \sin^2\pi x=1-\cos^2\pi x=(1+\cos\pi x)(1-\cos\pi x)$

$\lim_{x\to 1^+}1-\cos\pi x=2$

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$$\lim_{x\to -1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}} \xrightarrow{Hop}$$

$$\frac{\dfrac{2}{2\sqrt{2x+3}}-\dfrac{3}{2\sqrt{3x+4}}}{0+\dfrac{1}{3\sqrt[3]{x^2}}} \xrightarrow{x=-1} \frac{\dfrac{2}{2(1)}-\dfrac{3}{2(1)}}{\dfrac{1}{3}} = \frac{-\frac{1}{2}}{\frac{1}{3}} = \frac{-3}{2}$$

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$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}} \xrightarrow{Hop}$$

$$\frac{2-7\left(\dfrac{1}{2\sqrt{x}}\right)+0}{2-\dfrac{3}{2\sqrt{3x+1}}} \xrightarrow{x=1} \frac{2-\frac{7}{2}}{2-\frac{3}{4}} = \frac{\frac{-3}{2}}{\frac{5}{4}} = \frac{-6}{5}$$

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$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x} = \frac{1}{4} \qquad \frac{ab}{c}$$

$$\boxed{Hop}\ \frac{0+\dfrac{b}{2\sqrt{bx+c}}}{1} = \frac{1}{4} \xrightarrow{x=0} \frac{b}{\cancel{2}\sqrt{c}} = \frac{1}{\cancel{4}\,2} \rightarrow \frac{b}{\sqrt{c}} = \frac{1}{2} \rightarrow b = \frac{\sqrt{c}}{2}$$

$$a+\sqrt{b(0)+c}=0 \rightarrow a+\sqrt{c}=0 \rightarrow a=-\sqrt{c}$$

برای اینکه $\frac{0}{0}$ شود

$$\frac{ab}{c} = \frac{-\sqrt{c}\left(\frac{\sqrt{c}}{2}\right)}{c} = \frac{\frac{-c}{2}}{c} = -\frac{1}{2}$$

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۹

$$\lim_{x\to -2\pi} \frac{4+k\left[\frac{x}{\pi}\right]}{\sin x} = +\infty \qquad [-k]=?$$

$$\lim_{x\to (-2\pi)^+} \frac{4+k\left[\frac{x}{\pi}\right]}{\underbrace{\sin x}_{0^+}} = +\infty \longrightarrow 4-2k>0 \to 2k<4 \to k<2$$

$$\lim_{x\to (-2\pi)^-} \frac{4+k\left[\frac{x}{\pi}\right]}{\underbrace{\sin x}_{0^-}} = +\infty \longrightarrow 4-3k<0 \to 3k>4 \to k>\frac{4}{3}$$

$$\frac{4}{3}<k<2 \to [-k]=-2$$

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$$\lim_{x\to 8} \frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b}$$

$$\frac{0}{0} \sim 8a-b=0 \to 8a=b$$

$$\xrightarrow{\text{Hop}} \frac{\overset{8\cancel{a}}{\cancel{b}}\left(\dfrac{\frac{1}{3\sqrt[3]{x^2}}}{2\sqrt{2+\sqrt[3]{x}}}\right)}{\cancel{a}} = \frac{\overset{2}{\cancel{8}}\left(\frac{1}{12}\right)}{2(2)\cancel{4}} = \frac{1}{6}$$