۲۰ آفرین

مسانجانی / تکلیف شماره‌ی ۱۹

① $f(x) = 2\left[\frac{2-x}{2}\right] + a\left[\frac{x+2}{3}\right]$ $\quad \left[\frac{a}{3}\right] = ?$

$$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^-} f(x) \longrightarrow 2+0 = 4-a \rightarrow a = 2$$

$$\left[\frac{2}{3}\right] = 0$$

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② $$\lim_{x\to(\frac{\pi}{6})^-} [2\sin x - 1] = [0^-] = -1$$

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$$x < \frac{\pi}{6} \Rightarrow \sin x < \frac{1}{2} \rightarrow 2\sin x < 1 \rightarrow 2\sin x - 1 < 0$$

③ $$\lim_{x\to -\frac{1}{2}} [8x^3 - x] = \left[8\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)\right] = \left[-1 + \frac{1}{2}\right] = \left[-\frac{1}{2}\right] = -1$$

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④ $$\lim_{x\to(-\frac{1}{2})^-} \frac{10x - 5 + \left[\frac{3}{x^2}\right]}{16x - \left[-\frac{2}{x^2}\right]} = \lim_{x\to(-\frac{1}{2})^-} \frac{10x \overbrace{\cancel{-5+11}}^{6}}{16x + 8} = \frac{-5^- + 6}{-8^- + 8} = \frac{1}{0^-} = -\infty$$

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$$x < -\frac{1}{2} \rightarrow x^2 > \frac{1}{4} \rightarrow \frac{1}{x^2} < 4 \rightarrow \frac{3}{x^2} < 12$$

$$x < -\frac{1}{2} \rightarrow x^2 > \frac{1}{4} \rightarrow \frac{1}{x^2} < 4 \rightarrow \frac{2}{x^2} < 8 \rightarrow -\frac{2}{x^2} > -8$$

⑤ $$\lim_{x\to 1^+} \frac{\sin^2 \pi x}{[x] + \cos \pi x} = \lim_{x\to 1^+} \frac{\cancel{\sin^2 \pi x}}{\cancel{1+\cos \pi x}}$$

$$1 - \cos^2 \pi x = (1+\cos \pi x)(1-\cos \pi x)$$

$$\lim_{x\to 1^+} 1 - \cos \pi x = 2$$

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⑥

$$\lim_{x\to -1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}} \xrightarrow{HoP}$$

$$\frac{\dfrac{2}{2\sqrt{2x+3}}-\dfrac{3}{2\sqrt{3x+4}}}{0+\dfrac{1}{3\sqrt[3]{x^2}}} \xrightarrow{x=-1} \frac{\dfrac{2}{2(1)}-\dfrac{3}{2(1)}}{\dfrac{1}{3}} = \frac{-\frac{1}{2}}{\frac{1}{3}} = \frac{-3}{2}$$

⑦

$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}} \xrightarrow{HoP}$$

ب

$$\frac{2-7\left(\dfrac{1}{2\sqrt{x}}\right)+0}{2-\dfrac{3}{2\sqrt{3x+1}}} \xrightarrow{x=1} \frac{2-\frac{7}{2}}{2-\frac{3}{4}} = \frac{\frac{-3}{2}}{\frac{5}{4}} = \frac{-6}{5}$$

⑧

$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4} \qquad \frac{ab}{c}$$

$$\boxed{HoP}\ \frac{0+\dfrac{b}{2\sqrt{bx+c}}}{1}=\frac{1}{4} \xrightarrow{x=0} \frac{b}{2\sqrt{c}}=\frac{1}{4} \to \frac{b}{\sqrt{c}}=\frac{1}{2} \to b=\frac{\sqrt{c}}{2}$$

$$a+\sqrt{b(0)+c}=0 \to a+\sqrt{c}=0 \to a=-\sqrt{c}$$

برای اینکه $\frac{0}{0}$ شود

ب

$$\frac{ab}{c}=\frac{-\sqrt{c}\left(\frac{\sqrt{c}}{2}\right)}{c}=\frac{-\frac{c}{2}}{c}=-\frac{1}{2}$$

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$$\lim_{x\to -2\pi} \frac{4+k[\frac{x}{\pi}]}{\sin x} = +\infty \qquad [-k]=?$$

$$\lim_{x\to(-2\pi)^+} \frac{4+k[\frac{x}{\pi}]}{\underbrace{\sin x}_{0^+}} = +\infty \longrightarrow 4-2k>0 \rightarrow 2k<4 \rightarrow k<2$$

$$\lim_{x\to(-2\pi)^-} \frac{4+k[\frac{x}{\pi}]}{\underbrace{\sin x}_{0^-}} = +\infty \longrightarrow 4-3k<0 \rightarrow 3k>4 \rightarrow k>\frac{4}{3}$$

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$$\frac{4}{3}<k<2 \rightarrow [-k] = -2 \checkmark$$

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$$\lim_{x\to 8} \frac{b\sqrt{2+\sqrt[3]{x}} - 2b}{ax-b}$$

$$\frac{0}{0} \sim 8a-b=0 \rightarrow 8a=b$$

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$$\xrightarrow{\text{Hop}} \frac{\overset{8}{\cancel{b}}\left(\dfrac{\frac{1}{3\sqrt[3]{x^2}}}{2\sqrt{2+\sqrt[3]{x}}}\right)}{\cancel{a}} = \frac{\overset{2}{\cancel{8}}(\frac{1}{12})}{2(2)\cancel{4}} \checkmark = \frac{1}{6}$$