سارینا عابدینی

۱ـ

$$f(x) = 2\left[\frac{2-x}{2}\right] + a\left[\frac{x+2}{3}\right]$$

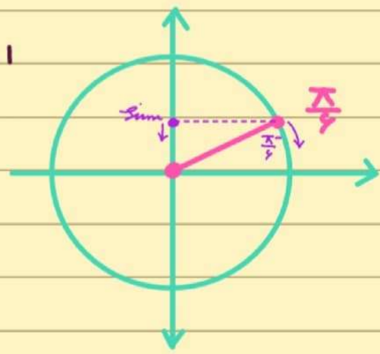
$$\lim_{x\to 2^+} 2\left[\frac{2-x}{2}\right] + a\left[\frac{x+2}{3}\right] = \cancel{2\left[\frac{4^-}{2}\right]}^{\,2} + \cancel{a\left[\frac{0^+}{3}\right]}^{\,0} = 2$$

$$\lim_{x\to 2^-} 2\left[\frac{2-x}{2}\right] + a\left[\frac{x+2}{3}\right] = \cancel{2\left[\frac{4^+}{2}\right]}^{\,4} + \cancel{a\left[\frac{0^-}{3}\right]}^{\,-a} = 4-a$$

$$\xrightarrow[\text{حد دارد}]{f(x) \text{ در } x=-2} 2 = 4-a \longrightarrow a = 2 \longrightarrow \left[\frac{a}{3}\right] = \left[\frac{2}{3}\right] = 0$$

۲ـ

$$\lim_{x\to \frac{\pi}{6}^-} [2\sin x - 1] = \left[2\times \frac{1}{2}^- - 1\right] = \left[1^- - 1\right] = \left[0^-\right] = -1$$



۳ـ

$$\lim_{x\to -\frac{1}{2}} [8x^3 - x] \xrightarrow{x\to -\frac{1}{2}^+} \left[-1^- + \frac{1}{2}^-\right] = -1$$

$$\xrightarrow{x\to -\frac{1}{2}^-} \left[-1^+ + \frac{1}{2}^-\right] = -1$$

حد چپ و راست برابر (-1) می باشند.

۴ـ

$$\lim_{x\to (-\frac{1}{2})^-} \frac{10x - 5 + \left[\frac{3}{x^2}\right]}{16x - \left[-\frac{2}{x^2}\right]} = \frac{-5-5+11}{-8^- + 8} = \frac{1}{0^-} = -\infty$$

$$x < -\frac{1}{2} \longrightarrow x^2 > \frac{1}{4} \longrightarrow \frac{1}{x^2} < 4 \longrightarrow \frac{3}{x^2} < 12 \longrightarrow \left[\frac{3}{x^2}\right] = 11$$

$$\longrightarrow \frac{1}{x^2} < 4 \longrightarrow -\frac{2}{x^2} > -8 \longrightarrow \left[\frac{-2}{x^2}\right] = -8$$

سارینا عابدینی

-۵

$$\lim_{x\to ۱^+}\frac{\sin^{۲}\pi x}{[x]+\cos\pi x}=\frac{۱-\cos^{۲}\pi x}{۱+\cos\pi x}=\frac{\cancel{(۱+\cos\pi x)}(۱-\cos\pi x)}{\cancel{(۱+\cos\pi x)}}=۱-\cos\pi x=۱-(-۱)=۲$$

-۶

$$\lim_{x\to -۱}\frac{\sqrt{۲x+۳}-\sqrt{۳x+۴}}{۱+\sqrt{x^{۳}}}\times\frac{\sqrt{۲x+۳}+\sqrt{۳x+۴}}{\sqrt{۲x+۳}+\sqrt{۳x+۴}}\times\frac{(\sqrt[۳]{x^{۲}}-\sqrt[۳]{x}+۱)}{(\sqrt[۳]{x^{۲}}-\sqrt[۳]{x}+۱)}=\frac{۲x+۳-۳x-۴}{۱+x}\times\frac{(\sqrt[۳]{x^{۲}}-\sqrt[۳]{x}+۱)}{\sqrt{۲x+۳}+\sqrt{۳x+۴}}$$

$$=(-۱)\times\frac{۳}{۲}=-\frac{۳}{۲}$$

-۷

$$\lim_{x\to ۱}\frac{۲x-۷\sqrt{x}+۵}{۲x-\sqrt{۳x+۱}}\xrightarrow{HOP}\frac{۲-\frac{۷}{۲\sqrt{x}}}{۲-\frac{۳}{۲\sqrt{۳x+۱}}}=\frac{۲-\frac{۷}{۲}}{۲-\frac{۳}{۴}}=-۱/۲$$

-۸

$$\lim_{x\to ۰}\frac{a+\sqrt{bx+c}}{x}=\frac{۱}{۴}\longrightarrow a+\sqrt{c}=۰\longrightarrow\sqrt{c}=-a \quad \mathrm{I}$$

$$\xrightarrow{HOP}\lim_{x\to ۰}\frac{\frac{b}{۲\sqrt{c}}}{۱}=\frac{b}{۲\sqrt{c}}=\frac{۱}{۴}\to ۴b=۲\sqrt{c}\to b=\frac{\sqrt{c}}{۲} \quad \mathrm{II}$$

$$\xrightarrow{\mathrm{I}\cap\mathrm{II}}\frac{a\times b}{c}=\frac{-\sqrt{c}\times\frac{\sqrt{c}}{۲}}{c}=\frac{-۱}{۲}$$

-۹

$$\lim_{x\to ۲\pi}\frac{۴+k\left[\frac{x}{\pi}\right]}{\sin x}\xrightarrow{x\to ۲\pi^+}\frac{۴-۲k}{۰^+}=+\infty\longrightarrow ۴-۲k>۰\to ۲>k \quad \mathrm{I}$$

$$\xrightarrow{x\to ۲\pi^-}\frac{۴-۳k}{۰^-}=+\infty\longrightarrow ۴-۳k<۰\to\frac{۴}{۳}<k \quad \mathrm{II}$$

$$\xrightarrow{\mathrm{I}\cap\mathrm{II}}\frac{۴}{۳}<k<۲\longrightarrow -۲<-k<-\frac{۴}{۳}\longrightarrow[-k]=-۲$$

-۱۰

$$\lim_{x\to ۸}\frac{b\sqrt{۲+\sqrt[۳]{x}}-۲b}{ax-b}=\frac{۰}{۸a-b}\longrightarrow ۸a-b=۰\longrightarrow ۸a=b$$

$$\xrightarrow{HOP}\lim_{x\to ۸}\frac{b\,\frac{\frac{۱}{۳}x^{-\frac{۲}{۳}}}{۲\sqrt{\sqrt[۳]{x}+۲}}}{a}=\frac{۸a\times\frac{۱}{۴}\times\frac{۱}{۳}}{۴a}=\frac{۱}{۶}$$