۲۰ آفرین

سارینا عابدینی

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$$f(x)=2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right]$$

$$\lim_{x\to 2^+} 2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right]=2\overset{2}{\cancel{\left[\frac{4^-}{2}\right]}}+a\overset{0}{\cancel{\left[\frac{0^+}{3}\right]}}=2$$

$$\lim_{x\to 2^-} 2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right]=2\overset{4}{\cancel{\left[\frac{4^+}{2}\right]}}+a\overset{-a}{\cancel{\left[\frac{0^-}{3}\right]}}=4-a$$

$$\xrightarrow[\text{حد طلب}]{x=-2 \text{ در } f(x)} 2=4-a \longrightarrow a=2 \longrightarrow \left[\frac{a}{3}\right]=\left[\frac{2}{3}\right]=0$$ ✓

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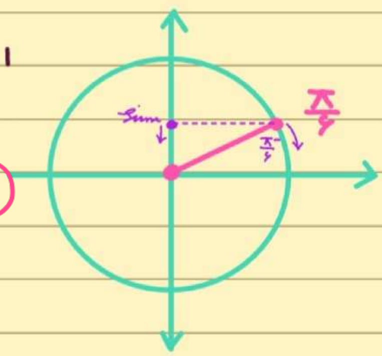
$$\lim_{x\to \frac{\pi}{6}^-}\left[2\sin x-1\right]=\left[2\times\frac{1}{2}^- -1\right]=\left[1^- -1\right]=\left[0^-\right]=-1$$ ✓



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$$\lim_{x\to -\frac{1}{2}}\left[8x^3-x\right]\xrightarrow{x\to -\frac{1}{2}^+}\left[-1^- +\frac{1}{2}^-\right]=-1$$

$$\xrightarrow{x\to -\frac{1}{2}^-}\left[-1^+ +\frac{1}{2}^-\right]=-1$$ ✓

حد چپ و راست برابر (-1) می باشند.

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$$\lim_{x\to (-\frac{1}{2})^-}\frac{10x-5+\left[\frac{3}{x^2}\right]}{16x-\left[-\frac{2}{x^2}\right]}=\frac{-5-5+11}{-8^-+8}=\frac{1}{0^-}=-\infty$$

$$x<-\frac{1}{2}\longrightarrow x^2>\frac{1}{4}\longrightarrow \frac{1}{x^2}<4\longrightarrow \frac{3}{x^2}<12\longrightarrow \left[\frac{3}{x^2}\right]=11$$ ✓

$$\longrightarrow \frac{1}{x^2}<4\longrightarrow -\frac{2}{x^2}>-8\longrightarrow \left[\frac{-2}{x^2}\right]=-8$$

سارینا عابدینی

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$$\lim_{x\to 1^+}\frac{\sin^2\pi x}{[x]+\cos\pi x}=\frac{1-\cos^2\pi x}{1+\cos\pi x}=\frac{\cancel{(1+\cos\pi x)}(1-\cos\pi x)}{\cancel{(1+\cos\pi x)}}=1-\cos\pi x=1-(-1)=2$$

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$$\lim_{x\to -1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x^3}}\times\frac{\sqrt{2x+3}+\sqrt{3x+4}}{\sqrt{2x+3}+\sqrt{3x+4}}\times\frac{(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}{(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}=\frac{2x+3-3x-4}{1+x}\times\frac{(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}{\sqrt{2x+3}+\sqrt{3x+4}}$$

$$=(-1)\times\frac{3}{2}=-\frac{3}{2}$$

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$$\lim_{x\to 1}\frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}}\xrightarrow{\text{HOP}}\frac{2-\frac{7}{2\sqrt{x}}}{2-\frac{3}{2\sqrt{3x+1}}}=\frac{2-\frac{7}{2}}{2-\frac{3}{4}}=-1/2$$

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$$\lim_{x\to 0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4}\to a+\sqrt{c}=0\to\sqrt{c}=-a \quad \mathrm{I}$$

$$\xrightarrow{\text{HOP}}\lim_{x\to 0}\frac{\frac{b}{2\sqrt{c}}}{1}=\frac{b}{2\sqrt{c}}=\frac{1}{4}\to 4b=2\sqrt{c}\to b=\frac{\sqrt{c}}{2} \quad \mathrm{II}$$

$$\xrightarrow{\mathrm{I}\cap\mathrm{II}}\frac{a\times b}{c}=\frac{-\sqrt{c}\times\frac{\sqrt{c}}{2}}{c}=\frac{-1}{2}$$

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$$\lim_{x\to 2\pi}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}\xrightarrow{x\to 2\pi^+}\frac{4-2k}{0^+}=+\infty\longrightarrow 4-2k>0\to 2>k \quad \mathrm{I}$$

$$\xrightarrow{x\to 2\pi^-}\frac{4-3k}{0^-}=+\infty\longrightarrow 4-3k<0\to\frac{4}{3}<k \quad \mathrm{II}$$

$$\xrightarrow{\mathrm{I}\cap\mathrm{II}}\frac{4}{3}<k<2\longrightarrow -2<-k<-\frac{4}{3}\longrightarrow[-k]=-2$$

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$$\lim_{x\to 8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b}=\frac{0}{8a-b}\longrightarrow 8a-b=0\longrightarrow 8a=b$$

$$\xrightarrow{\text{HOP}}\lim_{x\to 8}\frac{b\,\frac{\frac{1}{3}x^{-\frac{2}{3}}}{2\sqrt{\sqrt[3]{x}+2}}}{a}=\frac{8a\times\frac{1}{4}\times\frac{1}{3}}{4a}=\frac{1}{6}$$