۱)

$\lim_{x\to -2^+} f(x) = 2(1)+0 = 2 \qquad x>-2 \quad -x<2 \quad 2-x<4 \quad \frac{2-x}{2}<2 \quad x+2>0 \quad \frac{x+2}{3}>0$

$\lim_{x\to -2^-} f(x) = 2(2)-a \qquad x<-2 \quad -x>2 \quad 2-x>4 \quad \frac{2-x}{2}>2 \quad x+2<0 \quad \frac{x+2}{3}<0$

$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^-} f(x) \qquad 2=4-a \qquad a=2 \qquad \left[\frac{a}{3}\right]=\left[\frac{2}{3}\right]=0$

۳)

$\lim_{x\to -\frac{1}{2}} [8x^3-x] = [0^-] = -1 \qquad x(8x^2-1) \to \begin{cases} x=0 \\ x=\pm\frac{\sqrt{2}}{4} \end{cases}$

$-\frac{1}{2} \leftarrow \quad -\frac{\sqrt{2}}{4} \quad 0 \quad \frac{\sqrt{2}}{4} \qquad - \; + \; - \; +$

۲)

$\lim_{x\to (\frac{\pi}{6})^-} [2\sin x-1] = [0^-] = -1$

۴)

$\lim_{x\to(-\frac{1}{2})^-} \frac{10x-5+\left[\frac{3}{x^2}\right]}{14x-\left[-\frac{2}{x^2}\right]} = \frac{-5-5+11}{-8^- -(-8)} = \frac{1}{0^-} = -\infty \qquad x<-\frac{1}{2} \quad x^2>\frac{1}{4} \quad \frac{1}{x^2}<4 \quad \frac{3}{x^2}<12 \quad \frac{2}{x^2}<8 \quad \frac{-2}{x^2}>-8$

۵)

$\lim_{x\to 1^+} \frac{\sin^2\pi x}{[x]+\cos\pi x} = \lim_{x\to 1^+} \frac{\sin^2\pi x}{1+\cos\pi x} = \frac{\sin^2\pi}{1+\cos\pi} = \frac{1-\cos^2\pi}{1+\cos\pi} = \frac{(1-\cos\pi)(1+\cos\pi)}{1+\cos\pi} = 1-\cos\pi = 2$

۶)

$\lim_{x\to -1} \frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}} = \lim_{x\to -1} \frac{(\sqrt{2x+3}-\sqrt{3x+4})(\sqrt{2x+3}+\sqrt{3x+4})}{(1+\sqrt[3]{x})(\sqrt{2x+3}+\sqrt{3x+4})} = \lim_{x\to -1} \frac{2x+3-3x-4}{(1+\sqrt[3]{x})(\sqrt{2x+3}+\sqrt{3x+4})}$

$= \lim_{x\to -1} \frac{-(x+1)}{(1+\sqrt[3]{x})\times 2} = \lim_{x\to -1} \frac{-(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}{2} = -\frac{3}{2}$

۷)

$\lim_{x\to 1} \frac{2x-7\sqrt{x}+5}{2x-\sqrt{3x+1}} \xrightarrow{hop} \lim_{x\to 1} \frac{2-\frac{7}{2\sqrt{x}}}{2-\frac{3}{2\sqrt{3x+1}}} = \frac{-\frac{3}{2}}{\frac{5}{4}} = -\frac{6}{5}$

۸)

$\lim_{x\to 0} \frac{a+\sqrt{bx+c}}{x} \xrightarrow{hop} \lim_{x\to 0} \frac{\frac{b}{2\sqrt{bx+c}}}{1} = \frac{1}{4}$

چون مخرج صفر است ولی حاصل حد یک عدد است پس حد صورت صفر است.

$a+\sqrt{b(0)+c}=0 \qquad a=-\sqrt{c} \qquad \frac{b}{2\sqrt{c}}=\frac{1}{4} \qquad b=1 \qquad c=4 \to a=-2 \qquad \frac{ab}{c}=\frac{-2}{4}=-\frac{1}{2}$

۹)

$\lim_{x\to -2\pi} \frac{4+k\left[\frac{x}{\pi}\right]}{\sin x} = +\infty \qquad x\to -2\pi^+ : \frac{4+k[-2^+]}{0^+} = +\infty \qquad 4-2k>0 \qquad k<2$

$x\to -2\pi^- : \frac{4+k[-2^-]}{0^-} = +\infty \qquad 4-3k<0 \qquad k>\frac{4}{3} \qquad \frac{4}{3}<k<2 \to -2<-k<-\frac{4}{3} \qquad [-k]=-2$

۱۰)

$\lim_{x\to 8} \frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b} \xrightarrow{hop} \lim_{x\to 8} \frac{b\left(\frac{\frac{1}{3\sqrt[3]{x^2}}}{2\sqrt{2+\sqrt[3]{x}}}\right)}{a} = \frac{\frac{b}{48}}{a}$

حد صورت صفر است پس حد مخرج نیز صفر است.

$8a-b=0 \qquad b=8a$

$\xrightarrow{b=8a} \frac{1}{6}$