مینا جعفری پور — تکلیف ۱۹ — دوازدهم دختر B

۱
$f(x)=2\left[\frac{2-x}{2}\right]+a\left[\frac{x+2}{3}\right]=2\left[1-\frac{x}{2}\right]+a\left[\frac{x+2}{3}\right]=2\left((1)+\left[-\frac{x}{2}\right]\right)+a\left[\frac{x+2}{3}\right]$

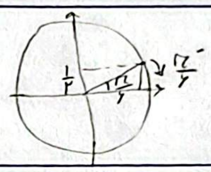
از براکت خارج شد

$\lim\limits_{x\to 2^+} f(x)=2(1+(0))+a(0)=2$

$\lim\limits_{x\to -2^-} f(x)=2(1+(1))+a(-1)=4-a$

$\Rightarrow \lim\limits_{x\to -2^+} f(x)=\lim\limits_{x\to -2^-} f(x)=\lim\limits_{x\to 2} f(x) \Rightarrow 2=4-a \Rightarrow a=2 \Rightarrow \left[\frac{a}{3}\right]=\left[\frac{2}{3}\right]=\boxed{0}$

۲
$\lim\limits_{x\to(\frac{\pi}{6})^-}[2\sin x-1]=\lim\limits_{x\to(\frac{\pi}{6})^-}[2\sin x]-1=0-1=\boxed{-1}$



$\sin x<\frac{1}{2}\Rightarrow 2\sin x<1$

۳
$\lim\limits_{x\to-\frac{1}{2}}[8x^3-x]=\lim\limits_{x\to-\frac{1}{2}}\left[8\left(\frac{-1}{8}\right)-\left(-\frac{1}{2}\right)\right]=\lim\limits_{x\to-\frac{1}{2}}\left[\underbrace{-1+\frac{1}{2}}_{-\frac{1}{2}}\right]=\boxed{-1}$

چون داخل براکت عدد صحیح نیست لازم به بررسی حالت های چپ و راست نیست

لذا $-\frac{1}{2}$ نیست

۴
$\lim\limits_{x\to(-\frac{1}{2})^-}\frac{10x-5+\left[\frac{3}{x^2}\right]}{16x-\left[\frac{-2}{x^2}\right]}=\frac{10x+6}{16x+8}=\frac{1}{0^-}=\boxed{-\infty}$

$x<-\frac{1}{2}\Rightarrow x^2>\frac{1}{4}\Rightarrow\frac{1}{x^2}<4\Rightarrow\begin{cases}\frac{3}{x^2}<12\Rightarrow\left[\frac{3}{x^2}\right]=11\\ \frac{-2}{x^2}>-8\Rightarrow\left[\frac{-2}{x^2}\right]=-8\end{cases}$

۵
$\lim\limits_{x\to1^+}\frac{\sin^2\pi x}{[x]+\cos\pi x}=\lim\limits_{x\to1^+}\frac{\sin^2\pi x}{1+\cos\pi x}=\lim\limits_{x\to1^+}\frac{\sin^2\pi x(1-\cos\pi x)}{\underbrace{(1+\cos\pi x)(1-\cos\pi x)}_{1-\cos^2\pi x=\sin^2\pi x}}=\lim\limits_{x\to1^+}(1-\cos\pi x)=\boxed{2}$

۶
$\lim\limits_{x\to-1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}}=\lim\limits_{x\to-1}\frac{\sqrt{2x+3}-\sqrt{3x+4}}{1+\sqrt[3]{x}}\times\frac{\sqrt{2x+3}+\sqrt{3x+4}}{\sqrt{2x+3}+\sqrt{3x+4}}\times\frac{(1+\sqrt[3]{x^2}-\sqrt[3]{x})}{(1+\sqrt[3]{x^2}-\sqrt[3]{x})}=\lim\limits_{x\to-1}\frac{-x-1}{1+x}\times\frac{3}{2}=\boxed{\frac{-3}{2}}$

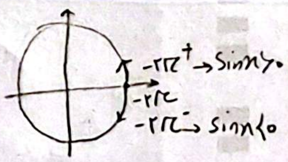
۷
$\lim\limits_{x\to1}\frac{2x-7\sqrt{x}+a}{2x-\sqrt{3x+1}}\overset{Hop}{=}\lim\limits_{x\to1}\frac{2-7\left(\frac{1}{2\sqrt{x}}\right)}{2-\left(\frac{1}{2\sqrt{3x+1}}\right)}=\frac{2-\frac{7}{2}}{2-\frac{1}{4}}=\boxed{\frac{-6}{7}}$

۸
$\lim\limits_{x\to0}\frac{a+\sqrt{bx+c}}{x}=\frac{1}{4}$ باوجود اینکه مخرج حد دارد پس ۰ است $\Rightarrow a+\sqrt{c}=0\Rightarrow\sqrt{c}=\underset{+}{-a}\Rightarrow c=a^2\ ,\ a<0$

$\overset{Hop}{=}\lim\limits_{x\to0}\frac{\frac{b}{2\sqrt{bx+c}}}{1}=\frac{b}{2\sqrt{c}}=\frac{1}{4}\Rightarrow2b=\sqrt{c}=-a\Rightarrow b=\frac{-a}{2}\Rightarrow\frac{ab}{c}=\frac{a\times\frac{-a}{2}}{a^2}=\boxed{\frac{-1}{2}}$

۹
$\lim\limits_{x\to-2\pi}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=+\infty$

$\begin{cases}\lim\limits_{x\to-2\pi^-}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=\lim\limits_{x\to-2\pi^-}\frac{4-3k}{0^-}=+\infty\Rightarrow4-3k<0\Rightarrow\frac{4}{3}<k\\ \lim\limits_{x\to-2\pi^+}\frac{4+k\left[\frac{x}{\pi}\right]}{\sin x}=\lim\limits_{x\to-2\pi^+}\frac{4-2k}{0^+}=+\infty\Rightarrow4-2k>0\Rightarrow2>k\end{cases}$



$\Rightarrow\frac{4}{3}<k<2\Rightarrow\frac{-4}{3}>-k>-2\Rightarrow[-k]=\boxed{-2}$

۱۰
$\lim\limits_{x\to8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b}=\lim\limits_{x\to8}\frac{2b-2b}{8a-b}$ حد متناهی است پس ۰ است $\Rightarrow 8a-b=0\Rightarrow b=8a\ (1)$

$=\lim\limits_{x\to8}\frac{b\sqrt{2+\sqrt[3]{x}}-2b}{ax-b}\times\frac{b\sqrt{2+\sqrt[3]{x}}+2b}{b\sqrt{2+\sqrt[3]{x}}+2b}=\lim\limits_{x\to8}\frac{b^2(\sqrt[3]{x}-2)}{ax-b}\times\frac{1}{4b}\times\frac{\sqrt[3]{x^2}+4+2\sqrt[3]{x}}{\sqrt[3]{x^2}+4+2\sqrt[3]{x}}=\lim\limits_{x\to8}\frac{b^2(x-8)}{\underset{(ax-8a)^{(1)}}{\cancel{ax-b}}}\times\frac{1}{4b}\times\frac{1}{12}$

$=\frac{b^2}{a}\times\frac{1}{4b}\times\frac{1}{12}=\frac{b}{48a}=\frac{8a}{48a}=\boxed{\frac{1}{6}}$