Date: Sub: تکلیف ۱۷ ثنا اصفهانی

سوال ۱

$$8\cos x - \tan^2 x = 1 \rightarrow 8\cos x = 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

$$\rightarrow \frac{1}{\cos^2 x} - 8\cos x = \frac{1 - 8\cos^3 x}{\cos^2 x} = 0$$

$$\rightarrow 1 - 8\cos^3 x = 0 \rightarrow \cos^3 x = \frac{1}{8} \rightarrow \cos x = \frac{1}{2} \rightarrow \begin{cases} x = \frac{\pi}{3} \\ x = 2\pi - \frac{\pi}{3} \end{cases}$$

← ۲ جواب در بازه $[0, 2\pi]$ دارد

سوال ۲

$$5\sin^2 x + 2\cos 3x + 2 = 5\sin^2 x + 2(\cos 3x + 1)$$

$$= 5\sin^2 x + 2\left(2\cos^2 \frac{3x}{2}\right) = 5\sin^2 x + 4\cos^2 \frac{3x}{2} = 0$$

$$\sin^2 x = 0 \longrightarrow x = k\pi \longrightarrow x = -\pi, 0, \pi$$

$$2\cos^2 \frac{3x}{2} = 0 \longrightarrow \cos\frac{3x}{2} = 0 \longrightarrow \frac{3x}{2} = k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{3}$$

$$\longrightarrow x = \frac{\pi}{3}, \pi, \frac{-\pi}{3}, -\pi$$

ریشه های مشترک π و $-\pi$ هستند بنابراین معادله دو جواب دارد

چون دو عبارت مثبت باهم جمع شده اند و حاصل صفر شده، هر کدام باید صفر باشند

سوال ۳

$$2\sin x\,(1-2\sin^2 x)+\sin x=1$$

$$2\sin x-4\sin^3 x+\sin x=1$$

$$4\sin^3 x-3\sin x+1=0 \xrightarrow{\sin x=t} 4t^3-3t+1=0$$

به ازای $t=-1$ صفر می‌شود

$$\Downarrow$$

$$(t+1)(2t-1)^2=0$$

$$t=-1 \qquad t=\frac{1}{2}$$

$$\sin x=-1 \longrightarrow x=2k\pi+\frac{3\pi}{2}$$

$$\sin x=\frac{1}{2} \longrightarrow x=2k\pi+\frac{\pi}{6}$$

$$x=2k\pi+\frac{5\pi}{6}$$

$$\begin{array}{l|l} 4t^3-3t+1 & t+1 \\ \hline 4t^3+4t^2 & 4t^2-4t+1 \\ \hline -4t^2-3t & \\ -4t^2-4t & \\ \hline t+1 & \\ t+1 & \\ \hline 0 & \end{array}$$

جواب‌ها در بازه $[0, 2\pi]$ ← $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ ← مجموعشان $= 2{,}5\pi$

سوال ۴

$$\frac{1}{\sin(\frac{\pi}{2}+2x)}+\frac{1}{\cos(\frac{\pi}{2}+2x)}=\frac{1}{\cos 2x}-\frac{1}{\sin 4x}=0$$

$$=\frac{1}{\cos 2x}-\frac{1}{2\sin 2x\cos 2x}=\frac{1}{\cos 2x}\left(1-\frac{1}{2\sin 2x}\right)=\frac{1}{\cos 2x}\left(\frac{2\sin 2x-1}{2\sin 2x}\right)=0$$

$$\rightarrow 2\sin 2x-1=0\rightarrow \sin 2x=\frac{1}{2}\quad 2x=2k\pi+\frac{\pi}{6}\rightarrow x=k\pi+\frac{\pi}{12}$$

$$2x=2k\pi+\frac{5\pi}{6}\rightarrow x=k\pi+\frac{5\pi}{12}$$

جواب‌ها در بازه $[0,\pi] \leftarrow \frac{\pi}{12},\ \frac{5\pi}{12}$ ← اختلاف $=\frac{4\pi}{12}=\frac{\pi}{3}=\alpha$

$$\tan\frac{2\pi}{3}=-\sqrt{3}$$

سوال ۵

$$\cos(x+\frac{\pi}{4})=\frac{1}{\sqrt{3}}\rightarrow \underbrace{\cos(2x+\frac{\pi}{2})}_{-\sin 2x}=2\left(\frac{1}{\sqrt{3}}\right)^2-1=\frac{-1}{3}$$

$$-\sin 2x=\frac{-1}{3}\rightarrow \sin 2x=\frac{1}{3}$$

$$m(\cos x-\sin x)-3\sqrt{6}\times\frac{1}{3}=\sqrt{6}\rightarrow m(\cos x-\sin x)=2\sqrt{6}$$

به توان ۲ می‌رسانیم $\rightarrow m^2(\underbrace{1-\sin 2x}_{\frac{2}{3}})=24\rightarrow m^2=36\rightarrow m=\pm 6$ و $m=-6$ غ ق ق ✓

سوال ۶

$$\sin(x-\frac{\pi}{3}+\frac{\pi}{2})\cos(x-\frac{\pi}{3})=\cos^2(x-\frac{\pi}{3})=1$$

$$\rightarrow \cos(x-\frac{\pi}{3})=1\rightarrow x-\frac{\pi}{3}=2k\pi\rightarrow x=2k\pi+\frac{\pi}{3}$$

$$\rightarrow \cos(x-\frac{\pi}{3})=-1\rightarrow x-\frac{\pi}{3}=2k\pi+\pi\rightarrow x=2k\pi+\frac{4\pi}{3}$$

جواب‌ها در بازه $[0,2\pi] \leftarrow \frac{\pi}{3},\ \frac{4\pi}{3}$ ← ۲ جواب

سوال ۴

$$\sin x + \sqrt{3}\cos x = \sqrt{2} \xrightarrow{\div 2} \sin x \times \frac{1}{2} + \frac{\sqrt{3}}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\rightarrow \sin\left(x + \frac{\pi}{3}\right) = \sin\frac{\pi}{4} \xrightarrow{1} x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \rightarrow x = 2k\pi - \frac{\pi}{12}$$

$$\xrightarrow{2} x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \rightarrow x = 2k\pi + \frac{5\pi}{12}$$

جواب ها در بازه $[-\pi \;\; 2\pi]$ $\rightarrow$ $\frac{-\pi}{12}, \frac{5\pi}{12}, \frac{23\pi}{12}$ $\rightarrow$ مجموعشان $= \frac{27\pi}{12}$

سوال ۵

$$4\sin x(-\cos x) = -4\sin x\cos x = -2\sin 2x = 1$$

$$\sin 2x = \frac{-1}{2} \rightarrow 2x = 2k\pi + \frac{7\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12}$$

$$2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12}$$

بازه $[0 \;\; 2\pi]$ $\rightarrow$ $\frac{7\pi}{12}, \pi - \frac{\pi}{12}, 2\pi - \frac{\pi}{12}, \pi + \frac{7\pi}{12}$ $\rightarrow$ مجموع $= 5\pi$

سوال ۹

$$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8}$$

$$\rightarrow \cos^2\alpha\cos^2 2\alpha\cos^2 4\alpha=\frac{1}{64}\rightarrow \cos\alpha\cos 2\alpha\cos 4\alpha=\pm\frac{1}{8}$$

$$\xrightarrow[\alpha\neq k\pi]{\times\sin\alpha}\underbrace{\sin\alpha\cos\alpha}_{\frac{1}{2}\sin 2\alpha}\cos 2\alpha\cos 4\alpha \quad\cdots\quad \frac{1}{8}\sin 8\alpha \;\Big|\; \frac{1}{8}\sin 8\alpha=\frac{\pm\sin\alpha}{8}\rightarrow \sin 8\alpha=\pm\sin\alpha$$

$$\xrightarrow{1}\sin 8\alpha=\sin\alpha\rightarrow 8\alpha=2k\pi+\alpha\rightarrow \alpha=\frac{2k\pi}{7}\rightarrow \max=\frac{6\pi}{7}$$

$$8\alpha=2k\pi+\pi-\alpha\rightarrow\alpha=\frac{2k\pi}{9}+\frac{\pi}{9}\rightarrow\max=\frac{7\pi}{9}$$

$$\xrightarrow{2}\sin 8\alpha=\sin(\pi+\alpha)\rightarrow 8\alpha=2k\pi-\alpha\rightarrow\alpha=\frac{2k\pi}{9}\rightarrow\max=\frac{8\pi}{9}$$

$$8\alpha=2k\pi+\pi+\alpha\rightarrow\alpha=\frac{2k\pi}{7}+\frac{\pi}{7}\rightarrow\max=\frac{5\pi}{7}$$

میدانیم که $\alpha=\pi$ قابل قبول نیست زیرا $\leftarrow$ $(2)(2)(2)=\frac{1}{8}$ ✗

بنابراین max جواب ها $\frac{8\pi}{9}$ است.

سوال ۱۰

$$\tan 3x = \frac{1}{\tan x} = \cot x = \tan\left(\frac{\pi}{2} - x\right)$$

$$3x = k\pi + \frac{\pi}{2} - x \rightarrow x = \frac{k\pi}{4} + \frac{\pi}{8}$$

جواب‌ها در بازه $[\pi , 2\pi]$ $\leftarrow$ $\frac{9\pi}{8} , \frac{11\pi}{8} , \frac{13\pi}{8} , \frac{15\pi}{8}$ $\leftarrow$ مجموع $= 6\pi$