مهسا نجاتی / تکلیف شماره ی ۱۷

①

$8\cos x - \tan^2 x = 1 \qquad [0, 2\pi]$

$8\cos x = \frac{1}{\cos^2 x} \to \cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2} \to$ ۲ جواب

$\left\{2k\pi + \frac{\pi}{3}, 2k\pi - \frac{\pi}{3}\right\}$

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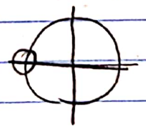
②

$\underbrace{5\sin^2 x}_{1-\cos^2 x} + 2\underbrace{\cos 3x}_{4\cos^3 x - 3\cos x} = -2 \qquad [-\pi, \pi]$

$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \to (\cos x + 1)\underbrace{(8\cos^2 x - 13\cos x + 7)}_{\Delta < 0} = 0$

$\cos x = -1 \to$  $\to$ ۲ جواب

۳)

$2\sin x\underbrace{\cos 2x}_{1-2\sin^2 x}+\sin x=1 \qquad [0, 2\pi]$

$2\sin x-4\sin^3 x+\sin x=1 \to 3\sin x-4\sin^3 x=1$

$\sin 3x=1 \to \sin 3x=\sin\frac{\pi}{2}$

$$\left.\begin{array}{l} 3x=2k\pi+\frac{\pi}{2} \\ 3x=2k\pi+\cancel{\pi-\frac{\pi}{2}}_{\frac{\pi}{2}} \end{array}\right\} x=\frac{2k\pi}{3}+\frac{\pi}{6}$$

k	0	1
x	$\frac{\pi}{6}$	$\frac{5\pi}{6}$

۲ جواب

$\frac{\pi}{6}+\frac{5\pi}{6}=\pi$

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۴)

$\frac{1}{\sin(\frac{\pi}{2}+2x)}+\frac{1}{\cos(\frac{\pi}{2}+2x)}=0 \qquad [0, \pi]$

$\frac{1}{\cos 2x}-\frac{1}{\cancel{\sin 4x}_{2\sin 2x\cos 2x}}=0 \to \frac{2\sin 2x-1}{2\sin 2x\cos 2x}=0 \to$

$2\sin 2x-1=0 \to \sin 2x=\frac{1}{2} \to \sin 2x=\sin\frac{\pi}{6}$

$$\left\{\begin{array}{l} 2x=2k\pi+\frac{\pi}{6} \quad =k\pi+\frac{\pi}{12} \\ 2x=2k\pi+\cancel{\pi-\frac{\pi}{6}}_{\frac{5\pi}{6}} \quad =k\pi+\frac{5\pi}{12} \end{array}\right.$$

k	0
x	$\frac{\pi}{12}$, $\frac{5\pi}{12}$

$\frac{5\pi}{12}-\frac{\pi}{12}=\frac{4\pi}{12}=\frac{\pi}{3}=\alpha \qquad \tan\frac{2\pi}{3}=-\sqrt{3}$

⑤ $\cos(x+\frac{\pi}{4})=\frac{1}{\sqrt{3}}$ ، $m(\cos x-\sin x)-3\sqrt{6}\,\underbrace{\sin 2x}_{2\sin x\cos x}=\sqrt{6}$

$\cos x\underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt2}{2}}-\sin x\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt2}{2}}=\frac{1}{\sqrt{3}}\to\frac{\sqrt{2}}{2}(\cos x-\sin x)=\frac{1}{\sqrt{3}}$

$\cos x-\sin x=\frac{2}{\sqrt{6}}$

$(\cos x-\sin x)^2=1-2\cos x\sin x=\frac{4}{6}=\frac{2}{3}$

$2\cos x\sin x=\frac{1}{3}\to\cos x$

$m(\frac{2}{\sqrt{6}})-\cancel{3}\sqrt{6}(\frac{1}{\cancel{3}})=\sqrt{6}\to m\frac{2\sqrt{6}}{6}=2\sqrt{6}\to m=6$

⑥ $\sin(x+\frac{\pi}{6})\underbrace{\cos(x-\frac{\pi}{3})}_{\text{متمم}}=1\quad[0,2\pi]$

$\sin(x+\frac{\pi}{6})=\frac{1}{2}\to\sin(x+\frac{\pi}{6})=\sin\frac{\pi}{6}$

$$\begin{cases}x+\cancel{\frac{\pi}{6}}=2k\pi+\cancel{\frac{\pi}{6}}\to x=2k\pi\\ x+\frac{\pi}{6}=2k\pi+\underbrace{\pi-\frac{\pi}{6}}_{\frac{5\pi}{6}}\to x=2k\pi+\frac{2\pi}{3}\end{cases}$$

k	0	1
x	0	2π
	$\frac{2\pi}{3}$	

۳ جواب

⑦ $\sin x+\sqrt{3}\cos x=\sqrt{2}$ ~~$\to\tan x=\sqrt{3}$~~ $[-\pi,2\pi]$

$\frac{1}{\sqrt{3+1}}$ ، $\frac{\sqrt{3+1}}{2}\sin(x+\frac{\pi}{3})=\sqrt{2}\to\sin(x+\frac{\pi}{3})=\frac{\sqrt{2}}{2}$ $\sin\frac{\pi}{4}$

$$\begin{cases}x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4}\to x=2k\pi-\frac{\pi}{12}\\ x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4}\to x=2k\pi+\frac{5\pi}{12}\end{cases}$$

k	0	1
x	$\frac{-\pi}{12}$	$2\pi-\frac{\pi}{12}$
	$\frac{5\pi}{12}$	$2\pi+\frac{5\pi}{12}$ X

$\frac{-\pi}{12}+\frac{5\pi}{12}+2\pi-\frac{\pi}{12}=2\pi+\frac{3\pi}{12}=2\pi+\frac{\pi}{4}$

۸

$$4\sin x \sin\left(\frac{3\pi}{2} - x\right) = 1 \qquad [0, 2\pi]$$

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$

$$-4\sin x\cos x = -2\sin 2x = 1 \rightarrow \sin 2x = -\frac{1}{2}$$

$$\sin 2x = \sin\left(-\frac{\pi}{6}\right)$$

$$\begin{cases} 2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12} \\ 2x = 2k\pi + \cancel{\pi + \frac{\pi}{6}} \rightarrow x = k\pi + \frac{7\pi}{12} \end{cases}$$

$$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

k	0	1
x	$-\frac{\pi}{12}$ ✗	$\frac{11\pi}{12}$
	$\frac{7\pi}{12}$	$\frac{19\pi}{12}$

$$\frac{7\pi}{12} + \frac{11\pi}{12} + \frac{19\pi}{12} = \frac{37\pi}{12}$$

۹

$$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = \frac{1}{8} \qquad [0, \pi]$$

$$\frac{\sin^2\alpha \cdot 2\cos^2\alpha \cdot 2\cos^2 2\alpha \cdot 2\cos^2 4\alpha}{\sin^2\alpha} = \frac{1}{8}$$

$$\frac{\cancel{\sin^2\alpha}\,\cancel{\cos^2\alpha}\cos^2 2\alpha\cos^2 4\alpha}{\sin^2\alpha} = \frac{1}{64} \rightarrow \frac{\sin^2 8\alpha}{\sin^2\alpha} = 1$$

$$\frac{\sin 8\alpha}{\sin\alpha} = \pm 1 \rightarrow \sin 8\alpha = \sin\alpha$$

$$\begin{cases} 8\alpha = 2k\pi + \alpha \rightarrow \alpha = \frac{2k\pi}{7} \\ 8\alpha = 2k\pi + \pi - \alpha \rightarrow \alpha = \frac{2k\pi + \pi}{9} \end{cases}$$

k	0	1	2	3	4
α	0	$\frac{2\pi}{7}$	$\frac{4\pi}{7}$	$\frac{6\pi}{7}$	$\frac{8\pi}{7}$ ✗
	$\frac{\pi}{9}$	$\frac{\pi}{3}$	$\frac{5\pi}{9}$	$\frac{7\pi}{9}$	π

$$A = \left\{0, \frac{\pi}{9}, \frac{2\pi}{7}, \frac{\pi}{3}, \frac{4\pi}{7}, \frac{5\pi}{9}, \frac{6\pi}{7}, \frac{7\pi}{9}, \pi\right\}$$

بیشترین مقدار

۱۰

$\tan 3x \tan x = 1 \qquad [\pi , 2\pi]$

$\tan 3x = \cot x \to \tan 3x = \tan(\frac{\pi}{2} - x)$

$3x = k\pi + \frac{\pi}{2} - x \to 4x = k\pi + \frac{\pi}{2} \to x = \frac{k\pi}{4} + \frac{\pi}{8}$

k	3	4	5	6	7
x	$\frac{7\pi}{8}$ ✗	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$\frac{9\pi}{8} + \frac{11\pi}{8} + \frac{13\pi}{8} + \frac{15\pi}{8} = 6\pi$