۱۸٫۷۵

مهسا نجاتی / تکلیف شماره‌ی ۱۷

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$8\cos x - \tan^2 x = 1 \qquad [0, 2\pi]$

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$8\cos x = \frac{1}{\cos^2 x} \to \cos^3 x = \frac{1}{8} \to \cos x = \frac{1}{2} \to$ ۲ جواب

$\left\{2k\pi + \frac{\pi}{3}, 2k\pi - \frac{\pi}{3}\right\}$

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$\underbrace{5\sin^2 x}_{1-\cos^2 x} + 2\underbrace{\cos 3x}_{4\cos^3 x - 3\cos x} = -2 \qquad [-\pi, \pi]$

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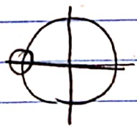
$8\cos^3 x - 5\cos^2 x - 6\cos x + 7 = 0 \to (\cos x + 1)(8\cos^2 x - 13\cos x + 7) = 0$

$\to \Delta < 0$

$\cos x = -1 \to$  $\to$ ۲ جواب

۳

$\cos 2x = 1 - 2\sin^2 x$ (فرمول طلایی)

۳ ۴ ۳

$2\sin x(1 - 2\sin^2 x) + \sin x = 1 \to 3\sin x - 4\sin^3 x = 1$

$\sin 3x = 1 \to 3x = 2k\pi + \frac{\pi}{2} \to x = \frac{2k\pi}{3} + \frac{\pi}{6} \qquad \xrightarrow{0 \le x \le 2\pi}$

$k = 0 \to x = \frac{\pi}{6}, \quad k = 1 \to x = \frac{5\pi}{6}, \quad k = 2 \to x = \frac{9\pi}{6}$

$S = \frac{\pi}{6} + \frac{5\pi}{6} + \frac{9\pi}{6} = \frac{15\pi}{6} = \boxed{\frac{5\pi}{2}}$

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$$۲\sin x \underbrace{\cos ۲x}_{۱-۲\sin^۲ x} + \sin x = ۱ \qquad [0, ۲\pi]$$

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$$۲\sin x - ۴\sin^۳ x + \sin x = ۱ \to ۳\sin x - ۴\sin^۳ x = ۱$$

$$\sin ۳x = ۱ \to \sin ۳x = \sin\frac{\pi}{۲}$$

$$\left.\begin{aligned} ۳x &= ۲k\pi + \frac{\pi}{۲} \\ ۳x &= ۲k\pi + \underbrace{\cancel{\pi - \frac{\pi}{۲}}}_{\frac{\pi}{۲}} \end{aligned}\right\} x = \frac{۲k\pi}{۳} + \frac{\pi}{۶}$$

k	0	۱
x	$\frac{\pi}{۶}$	$\frac{۵\pi}{۶}$

۲ جواب

$$\frac{\pi}{۶} + \frac{۵\pi}{۶} = \pi$$

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$$\frac{۱}{\sin(\frac{\pi}{۲} + ۲x)} + \frac{۱}{\cos(\frac{\pi}{۲} + ۲x)} = 0 \qquad [0, \pi]$$

$$\frac{۱}{\cos ۲x} - \frac{۱}{\underbrace{\cancel{\sin ۴x}}_{۲\sin ۲x \cos ۲x}} = 0 \to \frac{۲\sin ۲x - ۱}{۲\sin ۲x \cos ۲x} = 0 \to$$

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$$۲\sin ۲x - ۱ = 0 \to \sin ۲x = \frac{۱}{۲} \to \sin ۲x = \sin\frac{\pi}{۶}$$

$$\begin{cases} ۲x = ۲k\pi + \frac{\pi}{۶} = k\pi + \frac{\pi}{۱۲} \\ ۲x = ۲k\pi + \underbrace{\cancel{\pi - \frac{\pi}{۶}}}_{\frac{۵\pi}{۶}} = k\pi + \frac{۵\pi}{۱۲} \end{cases}$$

k	0
x	$\frac{\pi}{۱۲}$, $\frac{۵\pi}{۱۲}$

$$\frac{۵\pi}{۱۲} - \frac{\pi}{۱۲} = \frac{۴\pi}{۱۲} = \frac{\pi}{۳} = \alpha \qquad \tan\frac{۲\pi}{۳} = -\sqrt{۳}$$

⑤ $\cos(x+\frac{\pi}{4})=\frac{1}{\sqrt{3}}$

$m(\cos x-\sin x)-3\sqrt{6}\,\underbrace{\sin 2x}_{2\sin x\cos x}=\sqrt{6}$

$\cos x\underbrace{\cos\frac{\pi}{4}}_{\frac{\sqrt2}{2}}-\sin x\underbrace{\sin\frac{\pi}{4}}_{\frac{\sqrt2}{2}}=\frac{1}{\sqrt3}\rightarrow\frac{\sqrt2}{2}(\cos x-\sin x)=\frac{1}{\sqrt3}$

$\cos x-\sin x=\frac{2}{\sqrt6}$

$(\cos x-\sin x)^2=1-2\cos x\sin x=\frac{4}{6}=\frac{2}{3}$ ۲

$2\cos x\sin x=\frac{1}{3}\rightarrow\cos x$

$m(\frac{2}{\sqrt6})-\cancel{3}\sqrt6(\frac{1}{\cancel{3}})=\sqrt6\rightarrow m\frac{2\sqrt{6}}{6}=2\sqrt6\rightarrow m=6$

⑥ $\sin(x+\frac{\pi}{6})\underbrace{\cos(x-\frac{\pi}{3})}_{\text{متمم}}=1\quad[0,2\pi]$

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$\sin(x+\frac{\pi}{6})=\frac{1}{2}\rightarrow\sin(x+\frac{\pi}{6})=\sin\frac{\pi}{6}$

$$\begin{cases}x+\cancel{\frac{\pi}{6}}=2k\pi+\cancel{\frac{\pi}{6}}\rightarrow x=2k\pi\\ x+\frac{\pi}{6}=2k\pi+\underbrace{\pi-\frac{\pi}{6}}_{\frac{5\pi}{6}}\rightarrow x=2k\pi+\frac{2\pi}{3}\end{cases}$$

k	0	1
x	0	2π
	$\frac{2\pi}{3}$	

۳ جواب

⑦ $\sin x+\sqrt3\cos x=\sqrt2\rightarrow\tan x=\sqrt3\quad[-\pi,2\pi]$

$\frac{1}{|1|}\cancel{\sqrt{3+1}}\ \underset{2}{}\sin(x+\frac{\pi}{3})=\sqrt2\rightarrow\sin(x+\frac{\pi}{3})=\frac{\sqrt2}{2}\quad\sin\frac{\pi}{4}$

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$$\begin{cases}x+\frac{\pi}{3}=2k\pi+\frac{\pi}{4}\rightarrow x=2k\pi-\frac{\pi}{12}\\ x+\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{4}\rightarrow x=2k\pi+\frac{5\pi}{12}\end{cases}$$

k	0	1
x	$-\frac{\pi}{12}$	$2\pi-\frac{\pi}{12}$
	$\frac{5\pi}{12}$	$2\pi+\frac{5\pi}{12}$ X

$\frac{-\pi}{12}+\frac{5\pi}{12}+2\pi-\frac{\pi}{12}=2\pi+\frac{3\pi}{12}=2\pi+\frac{\pi}{4}$

(۸)

$4\sin x \sin\left(\frac{3\pi}{2} - x\right) = 1 \qquad [0, 2\pi]$

$-\cos x$

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$-4\sin x\cos x = -2\sin 2x = 1 \rightarrow \sin 2x = -\frac{1}{2}$

$\sin 2x = \sin\left(-\frac{\pi}{6}\right)$

$$\begin{cases} 2x = 2k\pi - \frac{\pi}{6} \rightarrow x = k\pi - \frac{\pi}{12} \\ 2x = 2k\pi + \pi + \frac{\pi}{6} \rightarrow x = k\pi + \frac{7\pi}{12} \\ \qquad \frac{7\pi}{6} \end{cases}$$

k	۰	۱
x	$-\frac{\pi}{12}$ ×	$\frac{11\pi}{12}$
	$\frac{7\pi}{12}$	$\frac{19\pi}{12}$

$\frac{7\pi}{12} + \frac{11\pi}{12} + \frac{19\pi}{12} = \frac{37\pi}{12}$

(۹)

$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha) = \frac{1}{8} \qquad [0, \pi]$

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$\frac{\sin^2\alpha \, 2\cos^2\alpha \, 2\cos^2 2\alpha \, 2\cos^2 4\alpha}{\sin^2\alpha} = \frac{1}{8}$

$\frac{\cancel{\sin^2\alpha}\,\cancel{\cos^2\alpha}\,\cos^2 2\alpha\,\cos^2 4\alpha}{\sin^2\alpha} = \frac{1}{64} \rightarrow \frac{\sin^2 8\alpha}{\sin^2\alpha} = 1$

$\frac{\sin 8\alpha}{\sin\alpha} = \pm 1 \rightarrow \sin 8\alpha = \sin\alpha$

$$\begin{cases} 8\alpha = 2k\pi + \alpha \rightarrow \alpha = \frac{2k\pi}{7} \\ 8\alpha = 2k\pi + \pi - \alpha \rightarrow \alpha = \frac{2k\pi + \pi}{9} \end{cases}$$

k	۰	۱	۲	۳	۴
α	۰	$\frac{2\pi}{7}$	$\frac{4\pi}{7}$	$\frac{6\pi}{7}$	$\frac{8\pi}{7}$ ×
	$\frac{\pi}{9}$	$\frac{\pi}{3}$	$\frac{5\pi}{9}$	$\frac{7\pi}{9}$	π

$A = \left\{0, \frac{\pi}{9}, \frac{2\pi}{7}, \frac{\pi}{3}, \frac{4\pi}{7}, \frac{5\pi}{9}, \frac{6\pi}{7}, \frac{7\pi}{9}, \pi\right\}$

بیشترین تعداد

(۶)

$$\cos\left(x-\frac{\pi}{3}\right)=\cos\left(\frac{\pi}{3}-x\right)$$

$$\frac{\pi}{3}-x+x+\frac{\pi}{6}=\frac{\pi}{2} \longrightarrow \cos\left(\frac{\pi}{3}-x\right)=\sin\left(x+\frac{\pi}{6}\right)$$

$$\sin^2\left(x+\frac{\pi}{6}\right)=1 \longrightarrow \sin\left(x+\frac{\pi}{6}\right)=\pm 1 \longrightarrow x+\frac{\pi}{6}=k\pi+\frac{\pi}{2}$$

$$x=k\pi+\frac{\pi}{3} \xrightarrow{0\leq x\leq 2\pi} k=0 \rightarrow x=\frac{\pi}{3} \;,\; k=1 \;,\; x=\frac{4\pi}{3}$$

معادله ۲ جواب در بازه دارد

(۱۰)

$$\tan 3x \tan x=1 \qquad [\pi, 2\pi]$$

$$\tan 3x=\cot x \rightarrow \tan 3x=\tan\left(\frac{\pi}{2}-x\right)$$

$$3x=k\pi+\frac{\pi}{2}-x \rightarrow 4x=k\pi+\frac{\pi}{2} \rightarrow x=\frac{k\pi}{4}+\frac{\pi}{8}$$

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k	۳	۴	۵	۶	۷
x	$\cancel{\frac{7\pi}{8}}$	$\frac{9\pi}{8}$	$\frac{11\pi}{8}$	$\frac{13\pi}{8}$	$\frac{15\pi}{8}$

$$\frac{9\pi}{8}+\frac{11\pi}{8}+\frac{13\pi}{8}+\frac{15\pi}{8}=6\pi$$

(۸)

$$\sin\left(\frac{3\pi}{2}-x\right)=-\cos x$$

$$4\sin x(-\cos x)=1 \longrightarrow -2(2\sin x\cos x)=1 \longrightarrow \sin 2x=-\frac{1}{2}$$

$$2x=2k\pi-\frac{\pi}{6} \;\text{یا}\; 2k\pi+\frac{7\pi}{6} \longrightarrow x=k\pi-\frac{\pi}{12} \;\text{یا}\; k\pi+\frac{7\pi}{12}$$

$$\xrightarrow{0\leq x\leq 2\pi} \begin{cases} k=0,1 \longrightarrow x=\frac{7\pi}{12}, \frac{19\pi}{12} \\ k=1,2 \longrightarrow x=\frac{11\pi}{12}, \frac{23\pi}{12} \end{cases}$$

$$S=\frac{7\pi}{12}+\frac{19\pi}{12}+\frac{11\pi}{12}+\frac{23\pi}{12}=\frac{60\pi}{12}=\boxed{5\pi}$$

$$1+\cos 2x = 2\cos^2 x$$

$$\begin{cases} 1+\cos 2\alpha = 2\cos^2\alpha \\ 1+\cos 4\alpha = 2\cos^2 2\alpha \\ 1+\cos 8\alpha = 2\cos^2 4\alpha \end{cases} \xrightarrow{?} 8\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{8}$$

$$\cos^2\alpha\,\cos^2 2\alpha\,\cos^2 4\alpha = \frac{1}{64} \xrightarrow{\sqrt{\ }} \cos\alpha\,\cos 2\alpha\,\cos 4\alpha = \frac{\pm 1}{8} \xrightarrow[\sin\alpha \neq 0]{\times \sin\alpha\ \bigstar}$$

$$\underbrace{(\sin\alpha\cos\alpha)}_{\sin 2\alpha}\cos 2\alpha\,\cos 4\alpha = \pm\frac{\sin\alpha}{8} \longrightarrow \frac{1}{2}\underbrace{(\sin 2\alpha\,\cos 2\alpha)}_{\frac{1}{2}\sin 4\alpha}\cos 4\alpha = \pm\frac{\sin\alpha}{8}$$

$$\frac{1}{2}\times\frac{1}{2}\underbrace{(\sin 4\alpha\,\cos 4\alpha)}_{\sin 8\alpha} = \frac{\pm\sin\alpha}{8} \longrightarrow \frac{1}{\cancel{8}}\sin 8\alpha = \pm\frac{\sin\alpha}{\cancel{8}}$$

$$\sin 8\alpha = \pm\sin\alpha \longrightarrow \begin{cases} 8\alpha = 2k\pi+\alpha \ \text{یا}\ 2k\pi+\pi-\alpha \\ 8\alpha = 2k\pi-\alpha \ \text{یا}\ 2k\pi+\pi+\alpha \end{cases}$$

$$\alpha = \frac{2k\pi}{7} \xrightarrow{MAX\ A} k=3 \longrightarrow x = \frac{6\pi}{7}$$

$$\alpha = \frac{2k\pi}{9} \xrightarrow{MAX\ A} k=4 \longrightarrow x = \frac{8\pi}{9}$$

$$\alpha = \frac{(2k+1)\pi}{7} \xrightarrow{MAX\ A} k=2 \longrightarrow x = \frac{5\pi}{7}$$

$$\alpha = \frac{(2k+1)\pi}{9} \xrightarrow{MAX\ A} k=3 \longrightarrow x = \frac{7\pi}{9}$$

$$\longrightarrow \boxed{MAX\ A = \frac{8\pi}{9}}$$

★ A نمی‌تواند برابر خود π باشد چون طبق ★ sinα را مخالف صفر در نظر گرفته‌ایم!