دوازدهم دختر B

بنام خدا

رضا ترکی
تکلیف شماره ۱۷۵

۱

$8\cos x - \tan^2 x = 1 \Rightarrow 1+\tan^2 x = 8\cos x \Rightarrow \frac{1}{\cos^2 x} = 8\cos x \Rightarrow 8\cos^3 x = 1$

$\Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2} \Rightarrow \cos x = \cos\frac{\pi}{3} \Rightarrow x = 2k\pi \pm \frac{\pi}{3} \xRightarrow{[0,2\pi]}$

$k=0 \longrightarrow x = \frac{\pi}{3}$ ✓

$k=1 \longrightarrow x = \frac{5\pi}{3}$ ✓

۲

$5\sin^2 x + 2\cos(3x) = -2 \longrightarrow \underbrace{5\sin^2 x}_{\text{نامنفی}} + \underbrace{2(\cos 3x + 1)}_{\text{نامنفی}} = 0 \Rightarrow$

$[-\pi,\pi]$

پس هر دو عبارت صفر بوده اند

$5\sin^2 x = 0 \longrightarrow \sin x = 0 \xrightarrow{[-\pi,\pi]} x = -\pi,\ x = \pi$

$\cos 3x = -1 \longrightarrow 3x = 2k\pi \pm \pi \longrightarrow x = \frac{2k\pi \pm \pi}{3} \longrightarrow$ جواب‌هایی صدق نمی‌کند

$-1 = \cos\pi$

$\Rightarrow$ معادله فقط ۲ جواب دارد: $-\pi$ ، π

راه ۲: $\boxed{\cos 3x + 1 = 2\cos^2\frac{3x}{2}}$

۳

$2\sin x\cdot\cos 2x + \sin x = 1 \Rightarrow \sin x(2\overbrace{\cos 2x}^{1-2\sin^2 x} + 1) = 1$

$\sin x(2 - 4\sin^2 x + 1) = 1 \Rightarrow -4\sin^3 x + 3\sin x = 1 \Rightarrow \sin 3x = 1 \Rightarrow \sin 3x = \sin\frac{\pi}{2}$

$\Rightarrow 3x = 2k\pi + \frac{\pi}{2} \longrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{6} \xRightarrow{[0,2\pi]}$

$k=0 \to x_1 = \frac{\pi}{6}$

$k=1 \to x_2 = \frac{5\pi}{6}$

$k=2 \to x_3 = \frac{9\pi}{6}$

$\Rightarrow x_1 + x_2 + x_3 = \frac{15\pi}{6} = \frac{5\pi}{2}$

۴

$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = \frac{-2\sin 2x + 1}{-2\sin 2x\cos 2x} = 0 \quad \sin 2x \neq 0,\ \cos 2x \neq 0$

$\sin 2x = \frac{1}{2} = \sin\frac{\pi}{6} \Rightarrow$

$x_1 = \frac{\pi}{12} \longleftarrow 2x = 2k\pi + \frac{\pi}{6} \to x = k\pi + \frac{\pi}{12}$

$x_2 = \frac{5\pi}{12} \longleftarrow 2x = 2k\pi + \frac{5\pi}{6} \to x = k\pi + \frac{5\pi}{12}$

$x_2 - x_1 = \frac{\pi}{3} = \alpha$

$\tan 2\alpha = \tan(\frac{2\pi}{3}) = -\tan\frac{\pi}{3} = -\sqrt{3}$

۵

$m(\cos x - \sin x) - 3\sqrt{6}\,\sin 2x = \sqrt{6}\ ,\ m = ?$

$\cos(x+\frac{\pi}{4}) = \frac{1}{\sqrt{3}} \Rightarrow \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \Rightarrow$

$\cos x - \sin x = \frac{2}{\sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$ ①

$\cos 2x$

$\cos(2x+\frac{\pi}{2}) = 2\cos^2(x+\frac{\pi}{4}) - 1 = \frac{2}{3} - 1 = \frac{-1}{3} \Rightarrow -\sin 2x = \frac{-1}{3} \Rightarrow$

$\sin 2x = \frac{1}{3}$ ②

$1, 2 \Rightarrow m \times \frac{\sqrt{6}}{3} - 3\sqrt{6} \times \frac{1}{3} = \sqrt{6} \Rightarrow \frac{\sqrt{6}}{3} m = 2\sqrt{6} \Rightarrow m = 6$ ج آخر

۶

$\cos(x-\frac{\pi}{3}) = \cos(\frac{\pi}{3} - x)$

$\sin(x+\frac{\pi}{6}) \cdot \cos(x - \frac{\pi}{3}) = 1$ → جمع دو زاویه متمم‌اند $\frac{\pi}{2}$

$\cos(\frac{\pi}{3} - x)$

$\cos^2(\frac{\pi}{3} - x) = 1$ → $\cos(\frac{\pi}{3} - x) = 1$, $\cos(\frac{\pi}{3} - x) = -1$

$\frac{\pi}{3} - x = 2k\pi \pm 2\pi$

$\frac{\pi}{3} - x = 2k\pi \pm \pi$

$\xrightarrow{[0, 2\pi]} \frac{\pi}{3} - x = k\pi \Rightarrow x = \frac{\pi}{3} - k\pi \to x = \frac{\pi}{3}\ ,\ x = \frac{4\pi}{3}$

۲ جواب

۷

$a\sin x + b\cos x = \frac{a}{|a|}\sqrt{a^2+b^2}\,\sin(x+\alpha)$, $\tan\alpha = \frac{b}{a}$

$\sin x + \sqrt{3}\cos x = \sqrt{2} \to 2\sin(x+\frac{\pi}{3}) = \sqrt{2}$

$\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

$\sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2} = \sin\frac{\pi}{4}$

$\to x + \frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{12}$

$x + \frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \to x = 2k\pi + \frac{5\pi}{12}$

$\xrightarrow{[-\pi, 2\pi]}$

$k=0 \to x_1 = \frac{-\pi}{12}\ ,\ x_2 = \frac{5\pi}{12}$

$k=1 \to x_3 = \frac{23\pi}{12}$

$\to x_1 + x_2 + x_3 = \frac{27\pi}{12} = \frac{9\pi}{4}$

۸

$4\sin x \sin(\frac{3\pi}{2} - x) = 1 \Rightarrow -4\sin x\cos x = 1 \Rightarrow$

$\frac{1}{2}\sin 2x = \frac{-1}{4} \Rightarrow \sin 2x = -\frac{1}{2} = \sin\frac{-\pi}{6}$

$\to 2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi + \frac{7\pi}{6} \to x = k\pi + \frac{7\pi}{12}$

$\xrightarrow{[0, 2\pi]} x = \frac{11\pi}{12}\ ,\ x = \frac{7\pi}{12}\ ,\ x = \frac{19\pi}{12}\ ,\ x = \frac{23\pi}{12}$

مجموع $\Rightarrow \frac{60\pi}{12} = 5\pi$

(۹)

$$(1+\cos\overset{۲\cos^۲\alpha-1}{۲\alpha})(1+\cos\overset{۲\cos^۲۲\alpha-1}{۴\alpha})(1+\cos\overset{۲\cos^۲۴\alpha-1}{۸\alpha})=\frac{1}{۸}$$

$$(۲\cos^۲\alpha)(۲\cos^۲۲\alpha)(۲\cos^۲۴\alpha)=\frac{1}{۸}\Rightarrow(\cos\alpha\cdot\cos۲\alpha\cdot\cos۴\alpha)^۲=\frac{1}{۶۴}$$

$$\xrightarrow[\div\sin\alpha]{\times\sin\alpha}\left(\frac{\sin\alpha\cos\alpha\cdot\cos۲\alpha\cdot\cos۴\alpha}{\sin\alpha}\right)^۲=\left(\frac{\frac{1}{۸}\sin۸\alpha}{\sin\alpha}\right)^۲=\frac{1}{۶۴}\Rightarrow\frac{1}{۶۴}\frac{\sin^۲۸\alpha}{\sin^۲\alpha}=\frac{1}{۶۴}$$

$$\sin^۲\alpha=\sin^۲۸\alpha\Rightarrow\frac{1-\cos۲\alpha}{۲}=\frac{1-\cos۱۶\alpha}{۲}\Rightarrow\cos۲\alpha=\cos۱۶\alpha\Rightarrow$$

$$۱۶\alpha=۲k\pi\pm۲\alpha\Rightarrow۸\alpha=k\pi\pm\alpha\to\begin{cases}۹\alpha=k\pi\to\alpha=\frac{k\pi}{۹}\\ ۷\alpha=k\pi\to\alpha=\frac{k\pi}{۷}\end{cases}\ A$$

$$\xrightarrow{[0,\pi]}\operatorname{Max}\alpha_A=\overset{k=۸}{\frac{۸\pi}{۹}}$$

(۱۰)

شرط صفر شدن cot, tan را چک کنید!
$\sin x\neq0$
$\cos x\neq0$

$$\tan۳x\cdot\tan x=1\longrightarrow\tan۳x=\frac{1}{\tan x}=\cot x$$

$$\tan۳x=\tan(\frac{\pi}{۲}-x)\longrightarrow۳x=k\pi+\frac{\pi}{۲}-x\Rightarrow۴x=k\pi+\frac{\pi}{۲}\xrightarrow{[\pi,۲\pi]}$$

$$x=\frac{k\pi}{۴}+\frac{\pi}{۸}$$

$$k=۴\to x=\frac{۹\pi}{۸},\ k=۵\to x=\frac{۱۱\pi}{۸},\ k=۶\to x=\frac{۱۳\pi}{۸},\ k=۷\to x=\frac{۱۵\pi}{۸}$$

$$\xRightarrow{\text{جمع}}\frac{۴۸\pi}{۸}=۶\pi$$