دوازدهم دهتد B

۲۰ آفرین

به نام خدا

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(۱)

$$8\cos x - \tan^2 x = 1 \Rightarrow 1+\tan^2 x = 8\cos x \Rightarrow \frac{1}{\cos^2 x} = 8\cos x \Rightarrow 8\cos^3 x = 1$$

$$\Rightarrow \cos^3 x = \frac{1}{8} \Rightarrow \cos x = \frac{1}{2} \Rightarrow \cos x = \cos\frac{\pi}{3} \Rightarrow x = 2k\pi \pm \frac{\pi}{3} \xrightarrow{[0,2\pi]}$$

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$$k=0 \to x = \frac{\pi}{3} \checkmark$$

$$k=1 \to x = \frac{5\pi}{3} \checkmark$$

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(۲)

$$5\sin^2 x + 2\cos(3x) = -2 \to \underbrace{5\sin^2 x}_{\text{نامنفی}} + \underbrace{2(\cos 3x + 1)}_{\text{نامنفی}} = 0 \Rightarrow$$

$[-\pi,\pi]$

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پس هر دو عبارت صفر بوده اند

$$5\sin^2 x = 0 \to \sin x = 0 \xrightarrow{[-\pi,\pi]} x = -\pi,\ x = \pi$$

$$\cos 3x = -1 \to 3x = 2k\pi \pm \pi \to x = \frac{2k\pi \pm \pi}{3} \to$$ جوابهای صدق نمی کند

$\cos\pi$

معادله فقط ۲ جواب دارد: $(-\pi)$ و (π)

۲ $\circ/\!/1$ $\quad \cos 3x + 1 = 2\cos^2\frac{3x}{2}$

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(۳)

$$2\sin x\cdot\cos 2x + \sin x = 1 \Rightarrow \sin x(2\overbrace{\cos 2x}^{(1-2\sin^2 x)} + 1) = 1$$

$$\sin x(2 - 4\sin^2 x + 1) = 1 \Rightarrow -4\sin^3 x + 3\sin x = 1 \Rightarrow \sin 3x = 1 \Rightarrow \sin 3x = \sin\frac{\pi}{2}$$

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$$\Rightarrow 3x = 2k\pi + \frac{\pi}{2} \to x = \frac{2k\pi}{3} + \frac{\pi}{6} \xrightarrow{[0,2\pi]} \begin{cases} k=0 \to x_1 = \frac{\pi}{6} \\ k=1 \to x_2 = \frac{5\pi}{6} \\ k=2 \to x_3 = \frac{9\pi}{6} \end{cases} \to x_1+x_2+x_3 = \frac{15\pi}{6} = \frac{5\pi}{2}$$

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(۴)

$$\frac{1}{\sin(\frac{\pi}{2}+2x)} + \frac{1}{\cos(\frac{\pi}{2}+4x)} = \frac{1}{\cos 2x} + \frac{1}{-\sin 4x} = \frac{-2\sin 2x + 1}{-2\sin 2x\cos 2x} = 0 \quad \begin{matrix}\sin 2x \neq 0 \\ \cos 2x \neq 0\end{matrix}$$

$$\sin 2x = \frac{1}{2} = \sin\frac{\pi}{6} \Rightarrow$$

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$$x_2 - x_1 = \frac{\pi}{3} = \alpha \Leftarrow \begin{cases} x_1 = \frac{\pi}{12} \leftarrow 2x = 2k\pi + \frac{\pi}{6} \to x = k\pi + \frac{\pi}{12} \\ x_2 = \frac{5\pi}{12} \leftarrow 2x = 2k\pi + \frac{5\pi}{6} \to x = k\pi + \frac{5\pi}{12} \end{cases}$$

$$\tan 2\alpha = \tan\left(\frac{2\pi}{3}\right) = -\tan\frac{\pi}{3} = -\sqrt{3}$$

(5)

$m(\cos x - \sin x) - 3\sqrt{6}\sin 2x = \sqrt{6}$, $m = ?$

$\cos(x+\frac{\pi}{4}) = \frac{1}{\sqrt{3}} \Rightarrow \frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \frac{\sqrt{2}}{2}(\cos x - \sin x) = \frac{1}{\sqrt{3}} \Rightarrow$

$\cos x - \sin x = \frac{2}{\sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$ (1)

$\cos 2x$

$\cos(2x+\frac{\pi}{2}) = 2\cos^2(x+\frac{\pi}{4}) - 1 = \frac{2}{3} - 1 = \frac{-1}{3} \Rightarrow -\sin 2x = \frac{-1}{3} \Rightarrow$

$\sin 2x = \frac{1}{3}$ (2)

$1, 2 \Rightarrow m\times\frac{\sqrt{6}}{3} - 3\sqrt{6}\times\frac{1}{3} = \sqrt{6} \Rightarrow \frac{\sqrt{6}}{3}m = 2\sqrt{6}$

$m = 6$

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(6)

$\cos(x-\frac{\pi}{3}) = \cos(\frac{\pi}{3}-x)$

$\sin(x+\frac{\pi}{6})\cdot\cos(x-\frac{\pi}{3}) = 1$ → جمع دو زاویه متمم‌اند

$\sin(x+\frac{\pi}{6}) = \cos(\frac{\pi}{3}-x)$

$\cos^2(\frac{\pi}{3}-x) = 1$ → $\cos(\frac{\pi}{3}-x) = 1$, $\cos(\frac{\pi}{3}-x) = -1$

$\frac{\pi}{3}-x = 2k\pi \pm 2\pi$

$\frac{\pi}{3}-x = 2k\pi \pm \pi$

$\xrightarrow{[0, 2\pi]}$ $\frac{\pi}{3}-x = k\pi \Rightarrow x = \frac{\pi}{3} - k\pi$ → $x = \frac{\pi}{3}$, $x = \frac{4\pi}{3}$

۲ جواب

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(7)

$\sin x + \sqrt{3}\cos x = \sqrt{2} \to 2\sin(x+\frac{\pi}{3}) = \sqrt{2}$

$a\sin x + b\cos x = \frac{a}{|a|}\sqrt{a^2+b^2}\sin(x+\alpha)$, $\tan\alpha = \frac{b}{a}$

$\tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

$\sin(x+\frac{\pi}{3}) = \frac{\sqrt{2}}{2} = \sin\frac{\pi}{4}$

$\to x+\frac{\pi}{3} = 2k\pi + \frac{\pi}{4} \to x = 2k\pi - \frac{\pi}{12}$

$x+\frac{\pi}{3} = 2k\pi + \frac{3\pi}{4} \to x = 2k\pi + \frac{5\pi}{12}$

$\xrightarrow{[-\pi, 2\pi]}$

$k=0 \to x_1 = \frac{-\pi}{12}$, $x_2 = \frac{5\pi}{12}$

$k=1 \to x_3 = \frac{23\pi}{12}$

$\to x_1 + x_2 + x_3 = \frac{27\pi}{12} = \frac{9\pi}{4}$

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(8)

$4\sin x\sin(\frac{3\pi}{2}-x) = 1 \Rightarrow -4\sin x\cos x = 1 \Rightarrow$

$\frac{1}{2}\sin 2x = \frac{-1}{4} \Rightarrow \sin 2x = -\frac{1}{2} = \sin\frac{-\pi}{6}$

$\to 2x = 2k\pi - \frac{\pi}{6} \to x = k\pi - \frac{\pi}{12}$

$2x = 2k\pi + \frac{7\pi}{6} \to x = k\pi + \frac{7\pi}{12}$

$\xrightarrow{[0, 2\pi]}$ $x = \frac{11\pi}{12}$, $x = \frac{7\pi}{12}$, $x = \frac{19\pi}{12}$, $x = \frac{23\pi}{12}$

مجموع $\Rightarrow \frac{60\pi}{12} = 5\pi$

$$(1+\cos 2\alpha)(1+\cos 4\alpha)(1+\cos 8\alpha)=\frac{1}{8}$$

($1+\cos2\alpha = 2\cos^2\alpha$, $1+\cos4\alpha = 2\cos^2 2\alpha$, $1+\cos8\alpha = 2\cos^2 4\alpha$; written above as $2\cos^2\alpha-1$, $2\cos^2 2\alpha-1$, $2\cos^2 4\alpha-1$)

$$(2\cos^2\alpha)(2\cos^2 2\alpha)(2\cos^2 4\alpha)=\frac{1}{8} \Rightarrow (\cos\alpha\cdot\cos 2\alpha\cdot\cos 4\alpha)^2=\frac{1}{64}$$

$$\xrightarrow[\div\sin\alpha]{\times\sin\alpha}\left(\frac{\sin\alpha\cos\alpha\cdot\cos2\alpha\cdot\cos4\alpha}{\sin\alpha}\right)^2=\left(\frac{\frac{1}{8}\sin 8\alpha}{\sin\alpha}\right)^2=\frac{1}{64}\Rightarrow\frac{1}{64}\frac{\sin^2 8\alpha}{\sin^2\alpha}=\frac{1}{64}$$

$$\sin^2\alpha=\sin^2 8\alpha\Rightarrow\frac{1-\cos2\alpha}{2}=\frac{1-\cos16\alpha}{2}\Rightarrow\cos2\alpha=\cos16\alpha\Rightarrow$$

$$16\alpha=2k\pi\pm2\alpha\Rightarrow 8\alpha=k\pi\pm\alpha$$

$$\to 9\alpha=k\pi\to\alpha=\frac{k\pi}{9}$$

$$\to 7\alpha=k\pi\to\alpha=\frac{k\pi}{7}$$

جواب A

$$\xrightarrow{[0,\pi]} Max_{A\ \text{جواب}}=\frac{8\pi}{9}\quad (k=8)$$

۱۰ — شرط صفر نشدن cot, tan را چک کنی! $\sin x\neq0$, $\cos x\neq0$

$$\tan 3x\cdot\tan x=1\longrightarrow\tan3x=\frac{1}{\tan x}=\cot x$$

$$\tan3x=\tan\left(\frac{\pi}{2}-x\right)\longrightarrow 3x=k\pi+\frac{\pi}{2}-x\Rightarrow 4x=k\pi+\frac{\pi}{2}\xrightarrow{[\pi,2\pi]}$$

$$x=\frac{k\pi}{4}+\frac{\pi}{8}$$

$$k=4\to x=\frac{9\pi}{8},\quad k=5\to x=\frac{11\pi}{8},\quad k=6\to x=\frac{13\pi}{8},\quad k=7\to x=\frac{15\pi}{8}$$

$$\xRightarrow{\text{جمع}}\frac{48\pi}{8}=6\pi$$